Non-linearity effects in the process of floods generation

M. FIORENTINO[a] & V. IACOBELLIS[b]

[a] *Dipartimento di Ingegneria e Fisica dell'Ambiente, Università della Basilicata, Contrada Macchia Romana, I-85100, Potenza, Italia - e-mail*: fiorentino@unibas.it
[b] *Dipartimento di Ingegneria delle Acque, Politecnico di Bari, via E. Orabona 4, I-70125, Bari, Italia - e-mail:* v.iacobellis@poliba.it

ABSTRACT

A recently proposed probability distribution of floods, based on the concept of variable runoff contributing area, was used to distinguish between the probability of occurrence of flood events generated by different runoff generation mechanisms. The statistical distribution of contributing area was characterized by means of the theoretical basis provided by the Horton and Dunne mechanisms. Non-linearity effects on the probability distribution were investigated, and, in particular, attention was paid to the possible non linear behavior in the runoff generation process, also with regard to the interactions with climate.

1 INTRODUCTION

The effects of non-linearity in the process of floods generation are analyzed by means of the theoretical framework provided in Iacobellis & Fiorentino (2000) where a derived probability distribution of floods, based on the concept of variable source area, was proposed. The partial area a contributing to the peak is considered as a main stochastic variable and its marginal distribution, which depends on both the spatial distribution and size of the storm and the prevailing runoff generating mechanism, is required. In particular, its expected value shows strong dependence on the climate, being lower in humid basins. This indicates that, consistently with the saturation excess (Dunne) process, on average only small portion of the area invested by the storm contributes to floods. Instead, in arid basins, where runoff is mainly generated by infiltration excess (Horton) mechanism, the flood peak source area, depending on the permeability of soils, is mainly limited by the portion covered by the storm and its distribution extends towards higher values. Such results are also consistent with comparison provided by Dooge (1997) which shows that, based on data from literature, lag time is usually greater for Dunne type than for Hortonian runoff, given any other geomorphological condition such as slopes and contributing area. Nevertheless, also arid basins with high permeability may show low values of the expected source area, indicating that small source areas are often responsible of runoff and significant subsurface flow may contribute to flood peak. In both cases we believe that non-linearity related to the eventual activation of the Horton or Dunne type mechanism, depending on storm severity, plays a crucial role in amplifying the skewness of flood series.

In the following we introduce such effects of non-linearity within the cited theoretical model, with the aim of better understand the physical processes involved in the rainfall-runoff transformation.

2 THE THEORETICALLY DERIVED ANNUAL MAXIMUM *CDF*

Within the theoretical model, the cumulative distribution function (*cdf*) of the peak direct streamflow Q is derived integrating, over the appropriate domain $R(q)$, the joint density function $g(u,a)$ of two main stochastic variables, namely the contributing area a and the peak unit runoff u.

$$G_Q(q) = prob[Q < q] = \iint_{R(q)} g(u,a)\,du\,da \tag{1}$$

$R(q)$ is found as the area within which the product of a times u is smaller than q.

The two variables u and a are considered as correlated, and their joint density function can be found considering the marginal distribution of one of them while the other one is conditional on the first. More explicitly, we used:

$$g(u,a) = g(u \mid a)\, g(a) \tag{2}$$

where $g(u \mid a)$ is the probability density function of u conditional on a (in the following synthetically indicated with u_a), while $g(a)$ is the marginal distribution of a.

The *cdf* of equation (1), by means of (2), can be expressed as:

$$G_Q(q) = \int_0^A \int_0^{\frac{q}{a}} g(u \mid a) g(a) du da \tag{3}$$

The flood peak is found as the sum of the peak direct streamflow and a base flow q_o

$$Q_p = Q + q_O \tag{4}$$

the *cdf* of the annual maximum values of Q_p is found within the hypothesis of Poisson distributed arrival process of independent flood events:

$$F_{Q_p}(q_p) = 1 - \frac{1}{T} = \exp\left\{-\Lambda_q \left[1 - G_{Q_p}(q_p)\right]\right\} \tag{5}$$

with recurrence interval T, in years, and Λ_q average annual number of independent peak streamflow events above q_o.

Introducing the average (in time and space) rainfall intensity $i_{a,\tau}$, the routing process is schematized by the equation

$$u_a = \xi \; (i_{a,\tau} - f_a) \tag{6}$$

in which ξ is a constant routing factor, u_a and $i_{a,\tau}$ are stochastic variables conditional on a, and the average water loss f_a is dependent on a by means of a relationship of the power type:

$$f_a = f_A (a/A)^{-\varepsilon'} \tag{7}$$

It is worth noting that equation (7) is based upon the hypothesis that the stochastic factors which affect the total amount of the hydrological losses at the event scale may be neglected when the focus is on the long term most probable state of the basin.

3 A TWO COMPONENT ANNUAL MAXIMUM *CDF*

Iacobellis & Fiorentino (2000) noticed, based on application of the described model on climatically heterogeneous basins, that in basins intensely vegetated, characterized by humid climate, the derived *cdf* of equation (5), in the following called 'one-component' *cdf*, is able to reproduce the observed annual flood maximum frequency, assuming small values of both the expected a and the average water losses f_A, as one would expect in a Dunne type behavior. Conversely, in arid climate the estimated average contributing area and water losses were higher, as in the typical Hortonian mechanism. Although the cited runoff generation mechanisms certainly involve many other and more complex aspects of the hydrologic response, here they are reduced to such simplified schemes. Then we denote the first type of behavior as of 'Dunne type', and the second one as the 'Horton type', analyzing the eventuality that a basin which on average contributes to runoff with small areas may, in particular cases, for precipitation of high intensity, contribute with large runoff source areas. The theoretical framework resumed in section 2, allows to distinguish between flood events generated by the so called Dunne or Horton type of runoff and the respective contributing area, as described in the following.

Depending on the runoff mechanism, the peak unit runoff has different characteristic and, in particular, the Dunne type of runoff, can be obtained by mean of a rainfall threshold, f_d, which is likely to assume low values in humid climate and higher values as much as the climate turns into arid. In such case we have:

$$u_{a,d} = \xi \, (i - f_{a,d}) \tag{8}$$

The second case is related to the Hortonian runoff generation mechanism and to the correspondent threshold $f_{a,h} > f_{a,d}$,

$$u_{a,h} = \xi \, (i - f_{a,h}) \tag{9}$$

In equations (8) and (9) we synthetically denoted with $u_{a,d}$ and $u_{a,h}$ the peak unit runoff conditional on the respective type of contributing area: $(u \mid a_d)$ and $(u \mid a_h)$. Both $f_{a,d}$ and $f_{a,h}$, if considered at the single event, should be treated as stochastic variables, nevertheless, they are here assumed as basin characteristic values, following the same type of scale relationship with a as in equation (7).

With regard to the variable contributing area a, one can distinguishes between areas generated by Dunne or Horton type of runoff, leading to the individuation of different statistical distribution, respectively $g(a_d)$ and $g(a_h)$ *pdfs*. The actual knowledge about these functions still needs deeper insights. We assumed that both are gamma distributed with only different value of the position parameter, respectively:

$$\alpha_d = r_d A / \beta, \text{ with } r_d = E[a_d]/A \tag{10}$$

and

$$\alpha_h = r_h A / \beta, \text{ with } r_h = E[a_h]/A \tag{11}$$

Furthermore, the results obtained with reference to basins with different climate ranging from arid to hyper-humid, indicate that the average a in humid basins is typically smaller, with values between 5 and 20% of total area, than in arid basins where it ranges from about 10 to 60% mainly depending on the permeable fraction of geologic units. These results were obtained assuming a single distribution of a and then, especially for basins with intermediate climatic features, they are representative of a mixed process compound of the overlapping mechanisms.

By means of the above defined quantities, it is possible to individuate two peak flow distributions:

$$G_{Q,d}(q) = \int_0^A \int_0^{\frac{q}{a}} g(u \mid a_d) g(a_d) du da_d \tag{12}$$

and

$$G_{Q,h}(q) = \int_0^A \int_0^{\frac{q}{a}} g(u \mid a_h) g(a_h) du da_h \tag{13}$$

which we recognize as two components of the same distribution of floods. Then, within the hypothesis of Poisson distributed arrivals of exceedances of both the introduced thresholds, the *cdf* of the flood annual maximum, can be obtained as:

$$F_{Q_p}(Q_{\max}) = \exp\left\{-\Lambda_d\left[1 - G_{Q,d}(Q_{\max})\right] - \Lambda_h\left[1 - G_{Q,d}(Q_{\max})\right]\right\} \tag{14}$$

where Λ_d and Λ_h are respectively mean annual number of Dunne and Horton type generated floods, and depend on $f_{A,d}$ and $f_{A,h}$ according to equations:

$$\Lambda_q = \Lambda_p \exp\left(-\frac{f_{a,d}{}^k}{E[i^k_{a,\tau}]}\right) \tag{15}$$

and

$$\Lambda_h = \Lambda_p \exp\left(-\frac{f_{a,h}{}^k}{E[i^k_{a,\tau}]}\right) \tag{16}$$

with the obvious condition that

$$\Lambda_q = \Lambda_d + \Lambda_h \tag{17}$$

The *cdf* in equation (14), as it will be shown in the application section by means of data from real basins, is able to reproduce flood peak distributions characterized by very high skewness and thick tail, depending on the mean annual number of Horton and Dunne type events and relative thresholds.

4 STUDY CASES

The above explained theory was applied to basins in Southern Italy showing high values of skewness. Here we report results relative to basins: Sinni at Valsinni, and Celone at Ponte Foggia San Severo. Their main features are reported in table 1. Their were chosen as representative of situations that may occur in climate respectively humid and arid or semiarid. In particular the second one, is a typical case of arid basin in which, despite of the climatic feature, the high degree of permeability is likely to produce a consistent threshold for Horton runoff over large areas, whose probability of occurrence is then limited to high return periods. Floods with greater probability of exceedance are, instead, triggered by less consistent contributing areas whose average becomes as low as less than 10% of total area. In this case it is probably not completely correct to speak of Dunne type runoff but it seems evident the presence of a process characterized by two different thresholds leading to the significant skewness which affects the flood's *cdf.*

SITE	N	$E[Q_p]$ (m^3/s)	Cv	Ca	Λ_q	q_o (m^3/s)
Sinni at Valsinni	22	555	0.56	2.42	19.1	45
Celone at Ponte Foggia San Severo	32	202	0.76	1.21	6.6	2.2

Table 1*a*. Measured flow statistics of the study cases. N, E [Q_p], Cv and Ca are length, sample average, coefficient of variation and skewness of the annual flood series; Λ_q: mean annual number of flood events; q_o: base flow.

SITE	A (km^2)	τ_A (h)	p_1 (mm/h)	n	Λ_p
Sinni at Valsinni	1140	5.6	23.13	0.405	21
Celone at Ponte Foggia San Severo	233	5.2	23.33	0.27	44.6

Table 1*b*. Geomorphological and rainfall characteristics of the study cases. A is basin area, τ_A basin lag-time; p_1 and n coefficient and exponent of the mean annual rainfall IDF (Intensity–Duration-Frequency) curve; Λ_p mean annual number of rainfall events.

In figure 1 and 2 are shown the obtained *cdfs*, together with the estimated plotting positions of the recorded data at the observed basins. Parameters used with the derived ‘one-component’ and ‘two-component’ *cdfs* are shown respectively in table 2*a* and 2*b*. The other parameters are chosen as Iacobellis and Fiorentino (2000) and assume the values k = 0.8, β = 4, $\varepsilon = \varepsilon'$ = 0.25 and ξ = 0.7. These are defined as: k, shape parameter of the Weibull distribution of rainfall intensity; β, shape parameter of the gamma distribution of a, a_d and a_h; ε and ε', exponents of the scale relationships with area of respectively mean areal rainfall intensity and average water loss; ξ, routing coefficient. In particular $f_{A,d}$, the lower threshold, was obtained as a function of Λ_q by equation (15) while the $f_{A,h}$ value was assumed equal to the mean value estimated on a number of arid basins in Southern Italy and, then, used for evaluation of Λ_h by means of equation (16). Consequently, Λ_d was obtained as difference between Λ_p and Λ_h, by equation (17). The estimated mean values of contributing area were found by calibration, fitting the derived *cdf* to the estimated plotting positions of recorded annual flood maximum series. In particular, the r values for the two basins were obtained by means of the one-component *cdf* in equation (5) while r_d and r_h by the two-component *cdf* of equation (14).

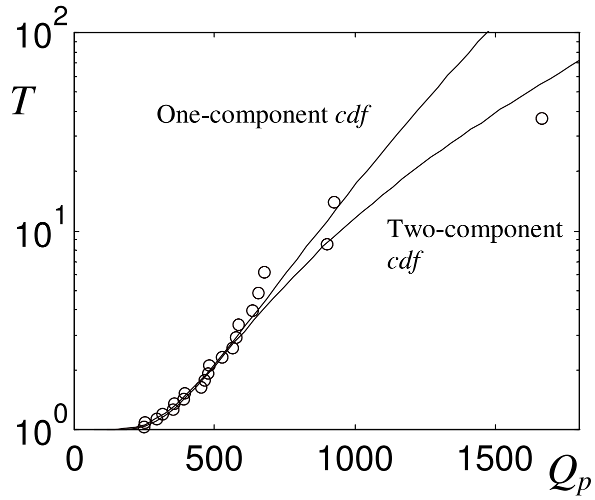

Figure 1. Sinni at Valsinni.

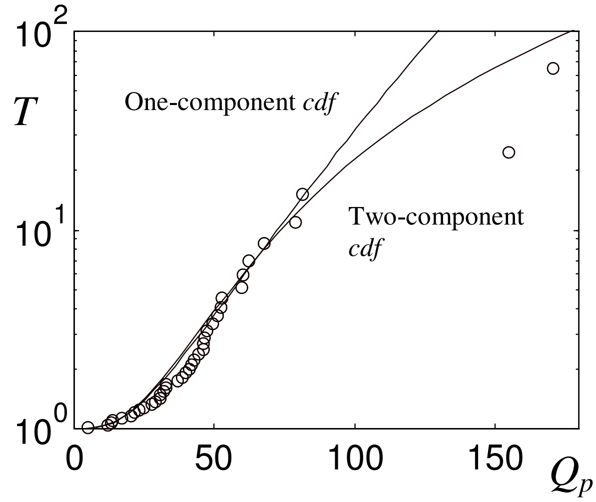

Figure 2. Celone at Ponte Foggia San Severo.

SITE	r	f_A (mm/h)	Λ_q
Sinni at Valsinni	0.20	0.07	19.1
Celone at Ponte Foggia San Severo	0.11	2.36	6.6

Table 2a. Estimated parameters by the one-component *cdf*.

SITE	r_d	r_h	$f_{A,d}$ (*mm/h*)	$f_{A,h}$ (*mm/h*)	Λ_d	Λ_h
Sinni at Valsinni	0.15	0.90	0.022	2.5	18.9	0.2
Celone at Ponte Foggia San Severo	0.10	0.70	0.581	2.5	6.5	0.1

Table 2b. Estimated parameters by the two-component *cdf*.

.

5 CONCLUSIONS

A number of consideration could be done by the light of the proposed model and we deem that particular remark can be given to the capability of the model to easily represent different physical behaviors of basins and their hydrological response to extreme rainfall events. Not the same satisfying evidence has been reached into the parameters estimation procedures which still complain the lack of suited collected data. In particular, with regard to the runoff generation mechanisms which control the statistical distributions of contributing area, still much needs to be confirmed and validated by real data. The same thresholds related to Horton and Dunne type of runoff should be treated in future research as key factors, to be observed and analyzed at the hillslope scale, but crucial for deeper understanding of the complex behavior of entire complex hydrological systems with reference to extreme events.

ACKNOWLEDGEMENTS

This work was supported by funds granted by the Consiglio Nazionale delle Ricerche – Gruppo Nazionale per la Difesa dalle Catastrofi Idrogeologiche. Project: “Va.Pi.-Ri.Vers.”.

REFERENCES

Dooge, J.C.I. Scale problems in hydrology, *Reflections on Hydrology: Science and Practice*, ed. Nathan Buras, American Geophysical Union, pp. 85-145, 1997.

Eagleson, P. S., Climate, soil, and vegetation 1. Introduction to water balance dynamics, *Water Resources Research*, **14**(5), 705-712, 1978.

Iacobellis, V. & Fiorentino, M. Derived distribution of floods based on the concept of partial area coverage with a climatic appeal, *Water Resources Research*, **36**(2), 469-482, 2000.