CONCEPTUALLY-BASED UNIVARIATE STOCHASTIC MODELLING OF RIVER RUNOFF

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1. INTRODUCTION

The reproduction of the hydrologic process of runoff is a fundamental part of the planning and management of water resources. The level of detail required in the simulation of the process depends on the objectives of the project. In the planning or management of reservoirs for irrigation or drinking water supply it is sufficient to deal with monthly runoff data. On the other hand, monthly data are inadequate for multipurpose reservoir (water supply and flood peak reduction) since at that scale information on the characteristics of runoff peaks is not significant. In this case, reproduction of data at resolution finer than monthly is needed.

Models for runoff characterisation can be synthetically subdivided in "essentially stochastic", generally univariate, and "essentially deterministic", generally bivariate. This classification leaves room for deterministic, physically-based, issues in stochastic models as well as for random error terms in the deterministic models.

In this paper univariate stochastic models will be discussed, with emphasis on possible physically-based issues in the identification and estimation of these models.

In the last three decades, univariate modelling of runoff series has been performed, to a great extent, using empirical approaches, looking for models that essentially can reproduce the stochastic structure of the time series under investigation. Models of the ARMA class (*Box and Jenkins*, 1970) have played a key role in this kind of approach, with variants depending on the existence of periodicity in runoff data. PARMA (periodic ARMA) models (*Tao and Delleur*, 1974) are the most notable variant of ARMA models for seasonal data. An early approach to seasonal data modelling by means of stationary ARMA models requires series deseasonalisation, while transformation of data is widely used to normalise the series. According to the Box-Jenkins approach, the objective in time series modelling is the *best fit* of the particular series analysed. Model type and order determination are only conditioned to the achievement of this objective (see *e.g. Rao et al.*, 1982).

Compared with this kind of approach, stochastic models based on physical considerations induce inherent reduction on the number of their possible types and orders. Reduction of the number of possible models to use are due to constraints caused by the conceptual hypotheses on which models themselves are based. This is not a limitation, indeed, for a number of reasons:

- conceptual model structure is founded on the information content of the observed series.

Therefore, conceptually-based models are inherently parsimonious, since they are as complex as is allowed by discrimination of different characters in the series observed;

- procedures of parameter estimation can be supported by conceptual hypotheses, provided the model structure is not too complex. On the other hand, objective statistical methods for parameter validation can be used when the model form is that of widely used stochastic models (*e.g.* ARMA models);

- in case of scarcity or lack of data, only conceptually-based models permit the transfer of information from similar basins in the context of the regional analysis;

1.1.Literature review of empirical stochastic models of runoff

Most of the work done in the last three decades in the field of the synthesis of streamflow sequences can be classified as part of a discipline named "Stochastic Hydrology", which was defined by the ASCE Committee on Surface-Water Hydrology (1965) as "the manipulation of statistical characteristics of hydrologic variables to solve hydrologic problems on the basis of stochastic properties of the variables". This definition is in substantial agreement with the main subdivision made by *Amorocho and Hart* (1964) of the hydrologic studies in *physical hydrology* (physical science research) and *stochastic hydrology* (system investigations).

Therefore, the latter field of investigation is characterised by an empirical approach to the problem, which ignores that "statistical and stochastic properties of hydrologic processes have definite physical causes and are amenable to explanation" (*Klemeš*, 1978). It must be recognised, however, that researches made in the field of the so-called "operational" (*Fiering*, 1966) or "utilitarian" (*Yevjevich*, 1991) hydrology contributed to the introduction into hydrology of the concept of stochastic process and of the basic methods of time series analysis (see *Kisiel*, 1969; *Lawrance and Kottegoda*, 1977; *Salas et al.*, 1980; *Bras and Rodriguez-Iturbe*, 1985, as examples of the achievements in the last decades).

Extensive reviews of the several classes of stochastic models proposed for operational hydrology can be found in *Lawrance and Kottegoda* (1977), *Franchini and Todini* (1981) and *Bras and Rodriguez-Iturbe* (1985). A closer look is devoted here to Autoregressive (AR) and mixed Autoregressive and Moving Average (ARMA) models (*Box and Jenkins*, 1970), that constitute the basic tool in today's time series modelling (see *Salas et al.*, 1980).

Autoregressive (or Markovian) processes are considered by *Kisiel* (1969) as one of the three classes (namely that of *transition-type processes*) of interest in hydrology, along with *counting-type processes* and with the *processes based on first and second moment functions*. Early applications of Autoregressive and Moving Average processes to hydrology are cited by *Kisiel* (1969, p.27-28). Particularly, it is striking that one of the most widely used models for monthly streamflow synthesis, the Lag-1 Multiseasonal Markovian model, was proposed by *Thomas and Fiering* in 1962. This model corresponds to an Autoregressive model of order 1 with periodic parameters, PAR(1), and, with limited improvements, still constitutes an high-level standard (*Noakes et al.*, 1984) among models built according to the "best fitting" principle. In the great number of applications of this model it is worth mentioning *Bacchi and Maione* (1983) who used it in the context of regional analysis of monthly streamflows.

Notable variants, mostly reviewed by *Lawrance and Kottegoda* (1977), were introduced over the simple lag-1 Autoregressive model (used, for instance by *Torelli and Tomasi*, 1976), mainly to try to account for the long memory effects displayed by streamflow time series, namely the *Hurst effect* (see *e.g. Klemeš*, 1974; *Lloyd*, 1967). In one of these variants (*Todini et al.*, 1979) correlation in the residuals was explicitly considered and modelled, while the Hurst coefficient was taken as a model parameter in the ARMA-Markov model proposed by *Lettenmaier and Burges* (1977).

The methodological approach proposed by *Box and Jenkins* (1970), based on the introduction of mixed Autoregressive and Moving Average (ARMA) processes, has given rise to a great number of applications (see *e.g. McKerchar and Delleur*, 1974, *Ubertini*, 1978, *Rao et al.*, 1982) and of theoretical studies (see *e.g. Salas et al.*, 1980 and *Bras and Rodriguez-Iturbe*, 1985, for an hydrological viewpoint, and *Piccolo*, 1990, for purely statistical aspects). Almost equivalent efforts were spent in "operational" hydrology on multivariate (intended as multisite) AR and ARMA models (see, *e.g.*, *Finzi et al.*, 1975, *Salas et al.*, 1980, *Salas et al.*, 1985, *Bartolini et al.*, 1988) mostly focused on the monthly scale. Other particular applications were made on daily runoff data with nonlinear autoregressive models (*Yakowitz*, 1973) or discrete ARMA (DARMA) models for precipitation associated to a linear model to produce runoff (*Chang et al.*, 1987).

Using a univariate "operational" approach, efficient reproduction of annual and monthly runoff can be achieved through models of the ARMA class. Particularly, the ARMA(1,1) model was shown (*O' Connell*, 1971) to be able of reproducing both short-term and long-term persistence effects observed on annual runoff series. Best performances in forecasting monthly runoff were shown by PAR(p) models, Periodic Autoregressive model with seasonally varying order (*Noakes et al.*, 1985) that compete with periodic ARMA (PARMA) models (*Tao and Delleur*, 1974) on the "operational" field, even if both classes are recently given conceptual-stochastic interpretation (*Salas and Obeysekera*, 1992). On the other hand, literature on daily runoff modelling reflects more closely the physical aspects characterising runoff at this scale, probably due to the fact that the intermittence and the patterns of floods are dominating aspects in the phenomenon and lead to a particular class of stochastic models.

Main criticisms that can be addressed to models lacking any connection with a physical interpretation of the phenomenon are: (*a*) The "best fitting" approach suggest to select the best model for each particular time series. This contrasts with the notion that the process does not depend on the location at which data are collected, at least within a homogeneous region; (*b*) when adopting model with periodic parameters for different series and a parameter is found not statistically significant, no hydrological validation can be undertaken to support the estimate; (*c*) If the length of the data set is insufficient, parameter estimation cannot be supported by any kind of additional consideration.

A most important point to focus is that if scientific literature somewhat reflects human needs, it is to remark that the "operational" approach to hydrological time series modelling seems to have reached more or less a level of satisfaction in "utilitarian" terms: more emphasis is thus needed in researches addressing the "physical bases" of hydrological processes as well as in the evaluation of quality of hydrologic data (*Yevjevich*, 1991). The topics which will be introduced in the sections below concern the "structural analysis" (in the sense intended by *Yevjevich*, 1991), of the runoff process over different scales of aggregation, consisting in the formulation of models of the process based on the recognition of its "structural" components.

1.2. Literature review of conceptually-based models

A common approach to description of the physical aspects related to runoff is *conceptualisation*, to be intended as the attempt to provide element of causal explanation in the choice of models. A *conceptual* approach to stochastic modelling of runoff, as opposed to the *empirical* approach, signifies "to relate variables based on the consideration of the physical processes acting upon the input variable(s) to produce the output variable(s)" (definition by *Clarke*, 1973). What qualifies conceptually-based models with respect to physically-based models is that the former are based on the observation of the runoff series, from which *essential* (in the sense of a lumped approach) characters of the process are highlighted. Physically-based models, on the other hand, are based on the analysis of all specific processes acting in the rainfall-runoff transformation (*e.g. Bathurst*, 1986). Therefore, observation of output (runoff) series is not needed for physically-based model building.

As widely commented on by *Klemeš* (1978), the greatest attention within the class of linear conceptual models has been paid to schemes in which the transformations operated by the watershed are reproduced by conceptual-deterministic schemes while the input is treated as a stochastic process. The conceptual representation of the watershed is essentially made up of a combination of (generally) linear reservoirs and linear channels.

Conceptual systems, built as combinations of linear reservoirs and linear channels subject to stochastic input, are equivalent to stochastic models. In particular, linear conceptual systems with stochastic input are equivalent to linear stochastic models, namely Autoregressive and Moving Average kind of models (ARMA). This point was dealt with in several papers, with different degrees of details, starting from the early papers by *Spolia and Chander* (1974) and *Moss and Bryson* (1974).

The above first approaches contain much of the variants one can introduce in conceptually-based modelling of runoff series, since *Spolia and Chander* (1974) consider linear reservoirs in series with stochastic or deterministic (bivariate case) input, while *Moss and Bryson* (1974) consider linear reservoirs and (implicitly) linear channels in parallel. From there, two important distinction are needed: the vast majority of the work done in the field of synthesis of the continuous runoff process has considered reservoirs in parallel, with or without linear channels while approaches considering linear reservoirs in series (*e.g. Klemeš and Boruvka*, 1975; *O'Connor*, 1976; and, as a particular case, *Pegram*, 1980) are mostly referred to flood forecasting models. The other consideration attains the nature of the input and requires further comments.

Both *Spolia and Chander* (1974) and *Moss and Bryson* (1974) move from the knowledge of the precipitation process. This gives the approach a bivariate character, regardless of the use of rainfall data as an observed (exogenous) input series or as a stochastic process of known properties. Indeed, this differentiation is substantial, producing ARX (AutoRegressive with eXogenous variable) models in the first case and ARMA models in the second case.

The conceptually-based stochastic analysis that will be discussed in this paper is related to univariate models, in which only the runoff data series is available for model building. This approach is adopted, in general, in the paper by *Pegram* (1980) and, more specifically, in the papers by *Salas and Smith* (1981) and *Salas et al.* (1981), who proved that a linear reservoir in parallel with a diversion, fed by a white noise input, is equivalent to an ARMA(1,1) stochastic model. *Salas and Obeysekera* (1992) later generalised the approach by *Salas et al.* (1981) showing that considering periodic-independent input the resulting class of stochastic models is that of PARMA models.

ARMA models require the input process to be continuous. In the case of intermittent input, as for time scales shorter than monthly, different stochastic models need to be selected as equivalent to the conceptual schemes. On these scales the key approach is based on Shot Noise processes (*e.g. Parzen,* 1962; *Bernier,* 1970) whose formulation is strictly referred to a conceptual scheme (see *e.g. Weiss,* 1977; *Koch,* 1985). A Shot Noise model not strictly based on a conceptual description of the net rainfall-runoff transformation was developed by *Treiber and Plate* (1975).

Interesting examples of alternative conceptually-based stochastic formulations are the ones by *Hino and Hasebe* (1981) and *Vandewiele and Dom* (1989), who started from the popular conceptual scheme with two parallel linear reservoirs (one accounting for a "fast" and the other for a "slow" response to effective precipitation) and followed more different ways to stochastic model identification and parameter estimation.

The number and the quality of the approaches for a conceptually-based stochastic analysis of streamflows indicate the interest for the conceptualisation of the runoff process. However, what really makes a difference among the approaches is not the originality of the proposed conceptual scheme but systematic balance between the physical considerations underlying a certain (more or less complex) conceptualisation and the information contained in the runoff data.

Conceptually-based stochastic model building is presented in this paper with the aim of setting up a framework that emulates the Box-Jenkins model building methodology, where the identification, estimation and validation steps are reformulated based on the use of the a-priori information on the physical process.

2. A RATIONALE FOR CONCEPTUALLY-BASED MODEL IDENTIFICATION ON DIFFERENT TIME SCALES

Univariate modelling of runoff based on physical considerations requires the *innovation* variable entering the model to produce the runoff process to have the meaning of *effective* (or *net*) *rainfall* (*i.e.* rainfall minus evapotranspiration) so that the model reproduces the net rainfall - runoff transformation.

In the alternative, bivariate, approach precipitation is introduced as a known process,so that the amount of available information increases but at the price of increasing model complexity, because the rainfall-net rainfall transformation must also be reproduced by the model. It is important, then, to assess the impact of the augmented model complexity on the number of parameters and on the model "identifiability" (see *Sorooshian and Gupta*, 1985; *Duan et al.*, 1992, among others).

The framework illustrated in this paper essentially relates stochastic models to linear conceptual models accounting for the transformation of the effective rainfall into runoff. A stochastic model for runoff at a given scale can be built according to the following hypotheses: (*a*) The input to the watershed system is a stochastic process, representing effective precipitation. Considerations on the nature of the input process determines the stochastic model type. (*b*) The conceptual model form at the scale of interest is obtained through a selection of linear conceptual elements producing substantial transformations on the input. This determines the stochastic model order, as will be discussed later.

Therefore, models with *stochastic input* and with *deterministic response,* identified based on physical considerations, are presented here in a framework of multicomponent univariate linear models. The formal structure of these stochastic models can be either that of ARMA or that of Shot Noise class of models depending on the structure of the net rainfall process which, in turn, is tied to the scale of aggregation.

2.1. Deterministic response modelling

2.1.1. PHENOMENOLOGICAL BASES OF STREAMFLOW PATTERNS

Observation of the runoff process patterns at different time scales allows one to recognise the effect of a number of distinct response components on the effective precipitation process, considered with reference to very different time spans. These runoff components are differentiated based on the different patterns that induce in the data series with the increase in the time scale considered.

To this end a first, general, distinction can be made between *direct runoff*, the fraction of runoff not reaching the groundwater table, and *baseflow*, or groundwater runoff, resulting from the decay of the groundwater storage in the subsoil. In the direct runoff, a distinction is usually made (see *e.g. Martinec*, 1985) between *surface runoff*, representing the amount of precipitation that does not infiltrates and covers only surface paths toward the outlet, and *subsurface runoff* (or *interflow*), produced by water paths that proceed partly into the soil layer. Surface runoff has the faster response to precipitation, varying with the basin size between a fraction of hour and a few days. Subsurface runoff becomes evident right after floods, with the typical change in the exponential decay of the flood hydrograph tail. Qualitatively, this component can have a response time ranging between several hours and several days.

Similarly, baseflow is better qualified by making distinction between components with different average response time, which depends on the hydrogeologic characteristics of the aquifers. This distinction is often evident when observing daily runoff data in the dry season, where a fast and a slow recession curve can usually be recognised in absence of precipitation.

Consider, for instance, basins of Central-Southern Italy, analysed by *Rossi and Silvagni* (1980), *Claps* (1990), *Murrone et al.* (1992a), *Claps et al.* (1993). It can be recognised that these basins are dominated by the geology of the Apennine mountains, presenting sometimes large fractured carbonate massifs with major aquifers at their base. Runoff deriving by the depletion of these deep aquifers can have lag time of a few years and induces long-term persistence effects in runoff series. The related runoff component will be designated as the *over-year* groundwater component.

Runoff from aquifers within geological non-carbonate formations and from overflow springs, which usually run dry by the end of the dry season, represents a distinct groundwater runoff term, with lag time of a few months (*over-month* groundwater component). At a given time, the discharge deriving from each of these components depends on the amount of the past effective precipitation relative to time interval of the same order of magnitude of the lag time of the component. Also important, as shown in the Appendix, is the variability of the net rainfall rate within the aggregation interval considered (*e.g.* periodic patterns within the annual span).

2.1.2. CHARACTERISTICS OF THE BASIN RESPONSE ON DIFFERENT TIME SCALES

The distinction of patterns observable in the runoff process tends to highlight as much as possible the origin of complexity in the net rainfall-runoff transformation, in view of the building of a simple yet efficient framework for stochastic model building. Consequently, model identification gives a considerable weight to a-priori considerations on the active components of the runoff process. Conceptual and stochastic model structures arising from the considerations made previously on the different aggregation scales will be discussed hereafter.

Let us select a sufficiently small time scale, say hourly, where all runoff components discussed before can be recognised from data. At this scale, groundwater and subsurface components can be regarded as the outlet of three linear reservoirs, implying an exponential-type response to the input. The same consideration would not apply to surface runoff, which is produced by the basin channel network response. Results from the geomorphologic IUH theory (*Rodriguez-Iturbe and Valdés*, 1979) assign a Gamma or Weibull function (*Troutman and Karlinger*, 1985) to the basin surface response to the effective rainfall.

By increasing the scale of aggregation of streamflow data (which decreases data resolution) surface runoff response can be approximated with an exponential function, which is the aggregated form of both functions cited above. With the same criterion, further increase of the scale reduces the detail with which the same exponential response of the surface runoff can be recognised. The same applies, with additional aggregation, to exponential forms of the subsurface runoff response.

This loss in detail of part of the response with the increase in the scale produces corresponding simplifications of the related conceptual and stochastic model structure. In other words, complexity of stochastic model structure must be minimised according to the detail in the information available. This rather basic concept, that has been often ignored (see *e.g. Sorooshian,* 1991), has been applied in the framework presented here according to the following rule: a runoff component is considered as the outlet of a linear reservoir if its characteristic time lag is of the same order of magnitude of the time scale unit considered; otherwise, if the lag is much smaller than the aggregation interval, it is considered as a simple diversion of the net rainfall.

2.2. Stochastic input modelling

2.2.1. INPUT SCHEMATIZATION IN THE CONCEPTUAL MODEL

The conceptual scheme considered for identification of stochastic model type and order is fed by a stochastic input that reaches the outlet through different paths. The issue now is: how precipitation subdivides into the four terms representing individual responses? The way we are going to approach the problem takes into account that individual responses derive from a lumped model, so that some approximations must obviously be assumed in the discussion.

In a first degree of schematization of the rainfall-soil interaction we assume that the mechanism of the direct runoff production is mainly of the Hortonian type, with saturation starting from the top soil layer. Moreover, due to the spatial variability of the infiltration capacity, the infiltration volume can be assumed proportional to precipitation.

Indicating with fP the infiltration in unsaturated soil layers, excess-rainfall $(1-f)P$ is only partly transformed in overland flow, to be conveyed downstream by the channel network as surface runoff. A portion of the excess-rainfall is transformed in subsurface flow, which follows, at least partly, paths within the saturated layers of the soil. Therefore, direct runoff components are both assumed proportional to total precipitation according to the relations:

$I_{d1}= \zeta_0(1-f)P$ *rainfall to be transformed in surface runoff*

$I_{d2}= \zeta_1(1-f)P$ *rainfall to be transformed in subsurface runoff*

where, neglecting variations in the volume stored in the soil in the interval considered, it can be written: $\zeta_0+\zeta_1 = 1$

Precipitation that infiltrates into the soil is partly subject to percolate toward the water table and partly to return to atmosphere as evapotranspiration E. Therefore, groundwater runoff components, are assumed proportional to the volume $fP-E$ according to relations:

$I_{g1} = \zeta_2(fP-E)$ *effective rainfall to be transformed in over-month groundwater runoff*

$I_{g2} = \zeta_3(fP-E)$ *effective rainfall to be transformed in over-year groundwater runoff*

where, again, $\zeta_2+\zeta_3 = 1$.

Linearity of the scheme just described is achieved only considering E proportional to P, which is a quite reasonable hypothesis in not very humid climates. The hypothesis of proportionality between E and P ($E = \Delta_E P$) allows us to use a univariate framework for evaluation of effective rainfall $I=P-E$ by elimination of the evapotranspiration term from the water balance. The input $I=P(1-\Delta_E P)$ is subdivided in surface and groundwater subsystems by means of recharge coefficients c_i:

$$I_0=c_0\,P(1-\Delta_E P); \quad I_1=c_1\,P(1-\Delta_E P); \quad I_3=c_2\,P(1-\Delta_E P); \quad I_3=c_3\,P(1-\Delta_E P)$$

which presupposes $c_0+c_1+c_2+c_3 = 1$ and the following relations with the former ζ_i parameters:

$$c_0 = \zeta_0\,\frac{P(1-f)}{P(1-\Delta_E)}\;; \qquad c_1 = \zeta_1\,\frac{P(1-f)}{P(1-\Delta_E)}$$

$$c_2 = \zeta_2\,\frac{P(f-\Delta_E)}{P(1-\Delta_E)}\;; \qquad c_3 = \zeta_3\,\frac{P(f-\Delta_E)}{P(1-\Delta_E)}$$

As should be clear by the above relations, subsystems in the conceptual model are arranged in parallel (see Fig. 1A) so that, due to linearity, the watershed system response turns out to be the sum of their individual responses. Main positive implication of this configuration is that model identification for aggregated time series becomes a straightforward task, as will be shown below.

Really, rainfall-net rainfall transformation is nonlinear, due to the effect of the threshold mechanism on the infiltration and the evapotranspiration processes, as well as for the effects of variability in the soil water content. On the other hand, linearity in the models allows one to take advantage of a great part of the literature in stochastic modelling, because many results of the autocorrelation analysis are based on hypotheses of stationarity and linearity. Moreover, multicomponent linear models allows apparent non linearity in the basin response to be reproduced through a piecewise linear schematization, which is efficient in terms of manageability and parsimony.

A linear conceptual scheme of watershed is a major or minor simplification of reality depending upon the time scale considered. This issue will be discussed with in mind the target of providing an *efficient* reproduction of the runoff process with respect to its use in water resources engineering. This is the reason why *conceptually-based* rather than *physically-based* frameworks are being discussed here.

Based on considerations made by *Moss and Bryson* (1974), *Salas and Obeysekera* (1992), *Claps et al.* (1993), among others, nonlinearities in the net rainfall-runoff transformation are essentially to be ascribed to the characteristics of the infiltration process. In this regard, it is worth remarking that recharge coefficients c_i indicate percentages of the net rainfall transformed in each of the runoff components.

Looking at the global net rainfall - runoff transformation, of great impact on the system nonlinearity is the variability of the ratio between direct and groundwater runoff, due to the influence of the soil moisture state and of the intensity of rainfall on the infiltration capacity. This influence increases with the decrease in the scale, due to the variability of conditions that can occur on soil saturation even between consecutive days.

Accordingly, relevance of variability in the recharge coefficients reduces increasing the scale. In particular, on the monthly scale this effect can be considered correlated with climatic seasons. On the annual scale, infiltration could be correlated to the amount of precipitation but essentially its variability is of minor significance.

In univariate streamflow simulation nonlinear models are seldom proposed, because of the computational burden involved and on the uncertainties related to the determination of the link between precipitation and infiltration in absence of the rainfall information. However, a way to set up this relationship could be get using the runoff level as an indicator of the rainfall amount, as considered by *Treiber and Plate* (1977) on the daily scale. These authors relate (linearly) parameters of the impulse response function to the runoff amount, even though the impulse response function adopted makes no use of a priori knowledge on the process.

2.2.2. CHARACTERISTICS OF THE PRECIPITATION PROCESS ON DIFFERENT TIME SCALES

A priori information on the input stochastic structure is needed for the selection of the runoff model type. To this end, stochastic models proposed for total precipitation are considered, to be used with reference to effective precipitation even though effective rainfall can have a quite different stochastic structure than that of total rainfall.

Considering that the rainfall stochastic structure simplifies with aggregation, the central limit theorem could be invoked in recognising that aggregation of precipitation from daily to annual basis produces processes approaching normality. The intermittent process becomes continuous, zero rainfall disappears and skewness decreases drastically. This will be discussed in detail hereafter.

On a hourly basis, precipitation shows a correlation structure that depends on the nature of the storm (see *e.g. Sirangelo and Versace*, 1990) and is an intermittent process. Correlation in the precipitation process is quite difficult to handle in univariate models (see comments by *Pegram*, 1980) even though it was considered, for instance, in the models proposed by *Treiber and Plate* (1977), *Pegram* (1980) and *Vandewiele and Dom* (1989) on scales ranging from hourly to weekly. *Cowpertwait and O'Connell* (1992) used, on data aggregated on a daily basis, a precipitation model based on the arrival of clusters of cells, namely the Neyman-Scott instantaneous pulse model.

Rainfall characteristics on daily or T-day scales are satisfactorily represented by uncorrelated processes, with Poisson-distributed occurrence of events (see *e.g. Eagleson*, 1978) and with either exponential or Gamma-distributed marks. Seasonality is incorporated in the occurrence rate of the Poisson process while intensity is generally deemed unaffected by the season. Seasonality in the Poisson model is generally achieved either by considering parameters that vary month by month or by using Fourier-type functions for continuous parameter variability within the year.

On the monthly scale, rainfall process is an independent periodic process. Lack of dependence in this process was shown by *Yevjevich and Karplus* (1973) among others. Depending on the underlying climate, the process may be continuous or of compound type, depending on the existence of a finite probability *P(0)* of zero rainfall. This last factor has been accounted for using a Bessel compound-type distribution (*e.g. Öztürk,* 1981) or considering *P(0)* as an additional parameter modifying the Box Cox transformation of the Normal distribution (*Legates,* 1991). Both distributions are able to reproduce the skewness displayed by monthly precipitation (see *e.g. Claps*, 1992). Either 12 parameter couples (both distributions are two-parameters) or a Fourier function allow consideration of seasonality in the probabilistic model.

On greater time scales, such as annual, where periodicity disappears along with wet and dry seasons, stationary and continuous probabilistic models (such as the Box-Cox transformation of the Normal distribution) can be used to reproduce the precipitation process.

2.3. Stochastic model structure

2.3.1. Conceptual and stochastic models for different scales

In the framework under discussion, elements of the conceptual model will be considered as linear reservoirs or zero-lag linear channels (diversions). Therefore, the most "complex" conceptual model that can be considered consists of four linear reservoirs in parallel, accounting for a slow and a fast component of groundwater runoff and a slow and a fast component of direct runoff.

The conceptual model structure is characterised by eight parameters (see Fig. 1A), the four *storage coefficients*, k_j, and the four *recharge coefficients*, c_j, of which only 7 are to be estimated given the volume continuity condition, $\Sigma c_j=1$.

The c_j coefficients represent the share of runoff produced, in average, by each component, and are determined by the net rainfall partition.

As introduced earlier, the stochastic nature of the univariate runoff models results from the structure of the input to the deterministic conceptual system. Due to the features of the precipitation and effective precipitation process discussed previously with reference to the hourly scale, a suitable stochastic model should be selected within the class of filtered Poisson processes (*Parzen*, 1962), best known as Shot Noise processes (*Bernier*, 1970). In particular, multicomponent Shot-Noise processes result from the particular conceptual model structure, with response function of the type:

$$h(s) = c_0\,\delta(0) + c_1 / k_1 e^{-s/k_1} + c_2 / k_2\, e^{-s/k_2} + c_3 / k_3\, e^{-s/k_3}$$

where the meaning of parameters will be clarified later.

Depending on the characteristic response time of the surface network, this model could be representative even for daily data. A simplification in the form of the surface runoff response arises, depending on the basin size, when data are aggregated on T days, with T considerably greater (say 2-3 times) than the mean lag time T_L of the surface runoff component. If $T >> T_L$, the surface runoff response can be considered impulsive (as a *Dirac* delta function) and the conceptual model can be rearranged as in Fig. 1B. With the scale T of the order of a few days, intermittence in the net rainfall process still produces a Shot-Noise stochastic model for runoff. In this case, the conceptual models has seven parameters and in any time interval the surface runoff is nothing but a fraction c_0 of the effective rainfall.

Further reduction in the complexity of the basin response structure occurs when data are considered at a scale sufficiently larger than the time lag of the interflow component. Aggregation may or may not determine an appreciable change in the net rainfall structure. However, to be short, one can consider monthly data and assume that at the monthly scale interflow can be incorporated in the direct runoff, proportional to effective precipitation (Fig. 2C). At the monthly scale, input is a quasi continuous process and drives to the selection of ARMA-type stochastic models (*e.g. Moss and Bryson*, 1974; *Salas and Smith*, 1981). In particular, monthly net rainfall is a periodic independent (quasi-continuous) stochastic process.

Since in the framework discussed here basin response is independent on the net rainfall characteristics, constant-parameter ARMA models with *periodic independent residual* or PIR-ARMA (*Claps et al.*, 1993) are selected for the monthly scale. Given the conceptual structure of the model in Fig. 2C, a PIR-ARMA(2,2) model is identified:

$$d_t - \Phi_1\, d_{t-1} - \Phi_2\, d_{t-2} = \varepsilon_t - \Theta_1\, \varepsilon_{t-1} - \Theta_2\, \varepsilon_{t-2}$$

with two AR and two MA parameters related, as will be clarified later, to the parameters of the conceptual model.

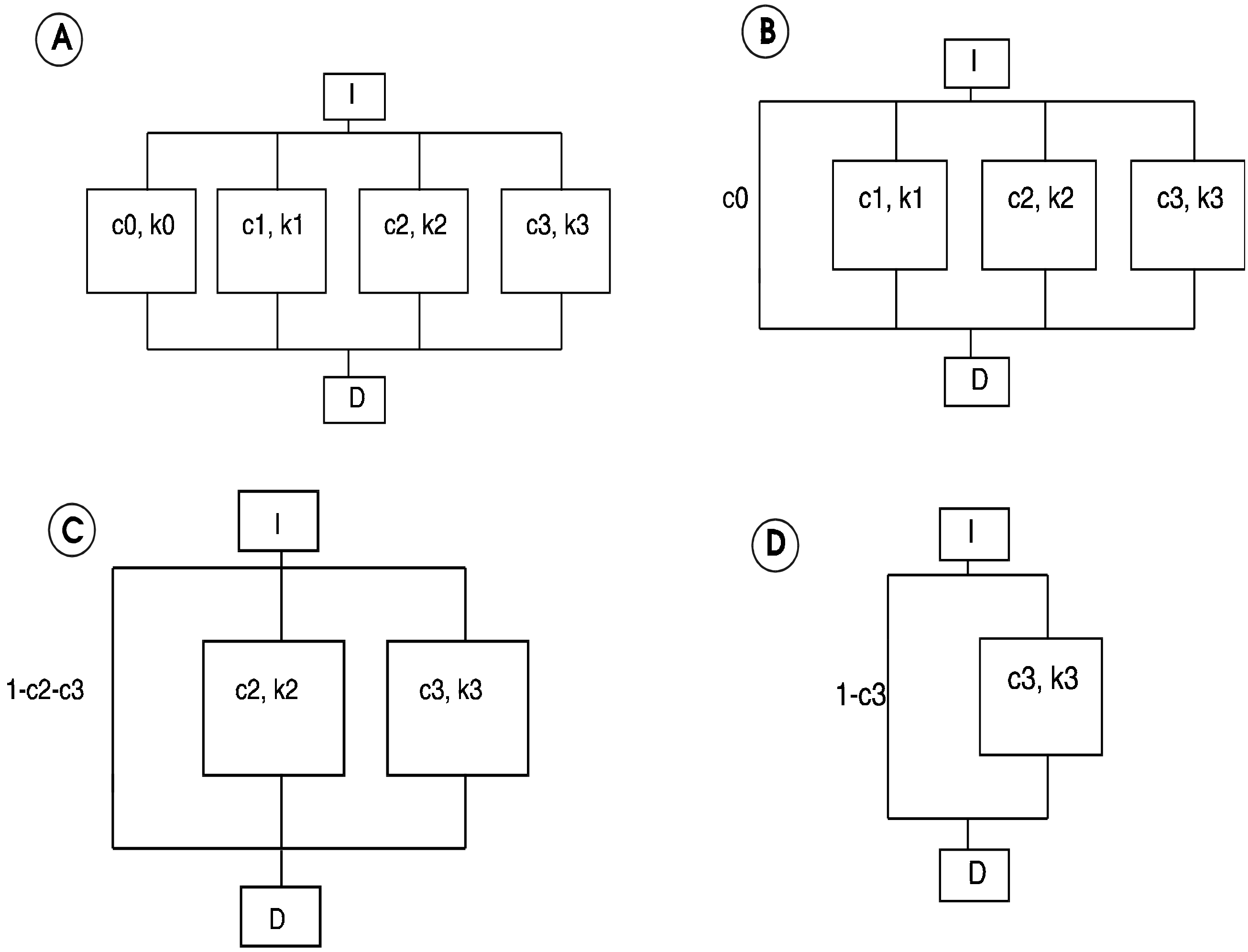

Figure 1. *Conceptual schemes underlying the transformation of effective rainfall I into runoff D at different aggregation scales.*

The usual aggregation step beyond monthly scale leads to the annual scale, where seasonal effects disappear and where the over-month groundwater component does not produce any effect. This is true particularly if data are considered aggregated on the hydrologic year, which starts at the end of the dry season (if two clearly distinct seasons can be identified). The reason is that at the end of a dry season all contribution by geological non-carbonate formations and from overflow springs can be considered to vanish. Therefore, water transfer between consecutive hydrologic years can be ascribed only to the deep groundwater runoff.

According to this scheme, at the annual scale, the only active transformation on effective rainfall is due to the deep groundwater linear reservoir, underlying the conceptual scheme of Fig. 1D. This conceptual model, subject to stationary input is equivalent to an ARMA(1,1) stochastic model (*Salas and Smith*, 1981):

$$d_t - \Phi_1\, d_{t-1} = \varepsilon_t - \Theta_1\, \varepsilon_{t-1}$$

2.3.2. SELECTION OF MODEL TYPE AND ORDER

The rules for conceptual model identification given in previous sections are strictly connected with the scale of aggregation, since the relative weight of the active components of runoff are related to the level of aggregation of the basic processes. Similar distinctions were made with reference to the structure of the input process. In this section, combination of conceptual model and input characteristics will be shown to lead, almost univocally, to identification of type and order of stochastic models.

Models for the annual time scale

Rossi and Silvagni (1980) first studied in detail the runoff process on the annual scale based on physical and climatic considerations, showing the utility of aggregating data on the hydrologic year and discussing two alternative stochastic models for the process reproduction.

The simplest model suggested by *Rossi and Silvagni* (1980) is a non-Gaussian independent model, to be used when annual runoff data are uncorrelated. This situation occurs for ephemeral streams, where the over-year groundwater component is missing and dry season runoff is almost negligible. With reference to data from rivers of Central and Southern Italy the probability distribution that better represent independent annual runoff was the cubic root Box-Cox transformation of the Normal distribution.

When noticeable autocorrelation is found in annual data, *Rossi and Silvagni* (1980) assumed it to be ascribed to an over-year groundwater component and substantiated with this consideration the validity of the ARMA(1,1) stochastic model (first proposed by *O'Connell*, 1971). *Salas and Smith* (1981) and *Salas et al.* (1981) formally derived the expression of the ARMA(1,1) model from a conceptual model of the basin response fed by uncorrelated Gaussian input. The expression of the ARMA(1,1) model is:

$$d_t - \Phi_1 \, d_{t-1} = \varepsilon_t - \Theta_1 \, \varepsilon_{t-1} \tag{1}$$

with d_t as the zero-mean runoff value at time t and ε_t as the zero-mean *residual* at time t. When derived from a conceptual model like this represented in Fig. 1D the ARMA(1,1) process presents a *restricted* parameter space, shown by *Salas et al.* (1981), which limits the range of values that stochastic parameters Φ and Θ can assume. This is due to the relations existing between conceptual and stochastic parameters, and provide an explanation to the range of parameter values that *O'Connell* (1971) found to allow ARMA(1,1) model to reproduce the *Hurst* effect.

It is worth mentioning that *Salas and Smith*, (1981) and *Salas et al.* (1981) consider the total precipitation (including evapotranspiration) as input to the conceptual system. Therefore, relations between stochastic and conceptual parameters as reported in *Claps and Rossi* (1991) for the strictly univariate case (with the effective rainfall representing the residual) are different, even though determine the same restricted parameter space.

Formal relations between conceptual and stochastic parameters were improved by *Claps and Rossi* (1992) and *Claps and Murrone* (1993) considering that effective rainfall does not enter the system as an impulse concentrated at the beginning of the time interval, as assumed by *Salas and Smith* (1981). This scheme is unrealistic for one-year interval due to the presence of within-year periodicity, and even for smaller intervals a rectangular input function is more appropriate.

The decision of the input function form has a considerable impact on the values of conceptual parameters resulting from stochastic parameter estimates and is further clarified in the Appendix.

The conceptual framework depicted in Fig. 1 is built in the hypothesis of presence of the over-year groundwater component. For ephemeral streams, the number of linear reservoirs for each scheme is reduced of 1 and the resulting model orders reduce accordingly (see Tab. 1).

Models for the monthly time scale

In the statistical approach to seasonal time series modelling, the presence of periodicity is usually the first problem to tackle. Stationarity is generally desired in order to use statistical tools developed for stationary stochastic variables or that provide more efficient results with stationary data.

Periodic processes are essentially dealt with using models with periodic parameters (*Tao and Delleur*, 1974) or deseasonalizing the series, for instance by monthly mean subtraction or by standardisation (*e.g. Delleur et al.*, 1976, *Salas et al.*, 1980). For similar reasons, a transformation is usually applied to data to achieve Normality.

In our viewpoint, both the above approach are inefficient. Deseasonalisation is a too strong transformation of the series: For instance, effects of the seasonal groundwater, having sub-annual evolution, are completely altered by the deseasonalisation. Moreover, deseasonalisation does not even eliminate periodicity in the autocorrelation function (*Tao and Delleur*, 1974). On the other hand, using models with periodic parameters generally involve overparametrisation, as will be discussed later.

In a physically-consistent approach to runoff modelling, the basic point to stress in this regard is that periodicity in runoff is ascribed essentially to periodicity in the input, which is also responsible for periodic patterns in the autocorrelation function. This consideration addresses to models with constant parameter and periodic input, which somewhat reflect a scheme proposed by *Pegram* (1980). The above discussion clarifies that the class of stochastic models considered for the monthly scale is that of the previously-introduced PIR-ARMA models.

Only for ephemeral streams, the simplest model for the monthly scale is a PIR-ARMA(1,1) (*Claps et al.*, 1993), formally identical to the ARMA(1,1) model introduced for the annual scale except for the nature of the residual. Therefore, the same restrictions as above apply to the parameter space.

The full model of monthly runoff resulting from the proposed framework is a PIR-ARMA(2,2)

$$d_t - \Phi_1\, d_{t-1} - \Phi_2\, d_{t-2} = \varepsilon_t - \Theta_1\, \varepsilon_{t-1} - \Theta_2\, \varepsilon_{t-2} \tag{2}$$

where $d_t = D_t - E[D_t]$ is the zero-mean runoff, ε_t is the zero-mean residual and Φ_1, Φ_2 and Θ_1, Θ_2 are autoregressive and moving average parameters, respectively. Stochastic parameters are explicitly connected with conceptual parameters c_3, c_2, k_3, k_2 (see relations A.30-A.37). The parameter space of this model (see Appendix) is restricted according to the constraints due to the conceptual meaning of parameters. The same restricted parameter space was found by *Spolia and Chander* (1974) with reference to a conceptual system with two linear reservoirs arranged in series.

To reproduce for a periodic variability in the recharge coefficients, stochastic models with periodic parameters can be used. Conceptually-based models with periodic parameters were recently recognised (*Salas and Obeysekera*, 1992) to be of the PARMA class (*Tao and Delleur*, 1974). In these models, all parameters are usually made seasonal, resulting in substantial overparametrisation (see *e.g. Kottegoda*, 1980, p.141), meaning excess of parameters compared to the available information.

In some cases (*Moss and Bryson*, 1974; *Hirsch*, 1979; *Salas and Obeysekera*, 1992) it has been suggested that the AR coefficients of PARMA models could be kept non-seasonal, recognising that the variability of the storage coefficient of the groundwater systems (which, in itself, should not be disregarded) is much less significant than the variability of the recharge coefficient in the different time intervals considered.

Even keeping constant the AR coefficients, however, the trade-off between difficulties and uncertainties in parameter estimation and accuracy in the conceptually-based transformation scheme gives, in our opinion, more credit to constant parameters models. In other words, a PARMA(2,2) model (as the periodic parameters equivalent to the PIR-ARMA(2,2) model of monthly runoff) where only periodic MA parameters are to be estimated still seems a too complex model compared to the refinement that introduces in the process simulation. This consideration is based also on the complexity of the estimation of MA parameters in PARMA models (*e.g. Jimenez et al.*, 1989).

Models for daily and T-day scale

The building of conceptually-based models of daily runoff requires some additional preliminary evaluations before deciding the model order, assuming, as said before, the model type as a Shot-Noise. Preliminary evaluations tend to determine if the characteristics of the surface runoff response can be recognised on the daily scale. For small basins (a few hundreds of km^2), daily aggregation generally does not allow to understand much of the transformation of the net rainfall in surface runoff. To disregard the effects of this transformation, for instance by considering the surface runoff transformation as a simple diversion operated by the basin on the net rainfall, one must be sure that this component does not produce any (even spurious) correlation effect, which can affect a correct estimation of parameters of the remaining components.

Murrone et al. (1992*a*) recognised that further aggregation of data on a scale unit of T days (with T>1) minimises the effect of the (aggregated) surface runoff on estimation of parameters of the interflow subsystem. The reason is that for data aggregated on T days the drainage network can be assumed to exercise no transformation on the net rainfall.

To apply this criterion, *Murrone et al.* (1992*b*) proposed to select the correct T-scale as the minimum one that allows the estimate of the interflow component parameters to stabilise. The range of T obtained for basins between 100 and 16000 km^2 is 1-7 days.

In a Shot Noise model, runoff *D* can be expressed, in continuous time, as

$$D(\tau) = \sum_{N(-\infty)}^{N(\tau)} Y_i \, h(\tau - \tau_i) \tag{3}$$

where $N(\tau)$ is the counting function of the Poisson process of occurrences. The response function *h(s)*, with $s = \tau - \tau_i$, is a linear combination of the response of all the conceptual elements (multicomponent response). For data aggregated at a suitable scale of T days, the surface impulse response is $c_0\,\delta(0)$,with $\delta(\cdot)$ as the Dirac delta function. Therefore, *h(s)* can be written as:

$$h(s) = c_0\,\delta(0) + c_1 / k_1 e^{-s/k_1} + c_2 / k_2\, e^{-s/k_2} + c_3 / k_3\, e^{-s/k_3} \tag{4}$$

and with c_i, k_i as parameters of the subsurface and the two groundwater conceptual elements. In case of lack of over-year groundwater, the response function becomes

$$h(s) = c_0\,\delta(0) + c_1 / k_1 e^{-s/k_1} + c_2 / k_2\, e^{-s/k_2} \tag{5}$$

If the resolution of data is such that the surface runoff response can be correctly recognised, an immediate extension of the above scheme can be used, with a four-component impulse response function:

$$h(s) = c_0 / k_0 e^{-s/k_0} + c_1 / k_1 e^{-s/k_1} + c_2 / k_2\, e^{-s/k_2} + c_3 / k_3\, e^{-s/k_3} \tag{6}$$

Tab. 1 summarises all the alternatives discussed above in terms of presence/absence of deep groundwater component and of possible refinements of models. As can be noted, the discussion has been limited to linear models.

Table 1. *Summary of possible alternative models identified on different time scales according to the conceptual framework represented in Fig. 1.*

scale	***most complex model***	***constant parameter model***	***least complex model***
annual	*ARMA(1,1)*	*ARMA(1,1)*	*independent, non-Gaussian*
monthly	*PARMA(2,2)*	*PIR-ARMA(2,2)*	*PIR-ARMA(1,1)*
T-day		*3-comp. Shot Noise*	*2-comp. Shot Noise*
hourly		*4-comp. Shot Noise*	*3-comp. Shot Noise*

3. CONCEPTUAL-STOCHASTIC MODEL BUILDING ON DIFFERENT TIME SCALES

When building a stochastic model, hydrological time series are generally regarded under the level of aggregation chosen for the purpose of simulation or forecasting. According to the usual practice, models aim to reproduce statistics that are strictly peculiar of the scale of interest. However, in a correct approach to streamflow analysis, models proposed and used for different time scales should be compatible in their structure.

Compatibility among scales comes by definition with conceptually-based models, because formal correspondences that can be established between conceptual and stochastic parameters allows one to use at whatever smaller scale the information related to conceptual parameters, but this issue was also addressed within the classical empirical stochastic analysis. For instance, *Kavvas et al.* (1977) analysed the runoff process on different scales under the time and the frequency domain. Based on this work, *Vecchia et al.* (1983), *Obeysekera and Salas* (1986) and *Bartolini and Salas* (1993) focused on the aggregation of periodic ARMA models, starting with pre-determined seasonal model and determining the autocovariance structure and the theoretical model of aggregated data. These authors obtained models consistent to each other, allowing one to use parameter estimates on a time scale to provide estimates for an aggregated time scale.

When building (conceptually-based) models in which the basic principle is that runoff is the sum of processes with different time response to precipitation, aggregation is given a more important role. Taken for granted that a conceptual parameter, *e.g.* a storage coefficient, does not change its value with the change of scale, the issue is whether there is a particular scale in which this parameter can be estimated most efficiently. In other words, does always the use of the maximum of information available (on the smallest scale) ensure the best knowledge of the process?

As an attempt to answer this question, *Claps et al.* (1993) first questioned the usual practice of estimating all parameters of a stochastic model on the data at the resolution for which the model is proposed, and suggested that parameter estimation on different time scales can give more efficient outcomes. In particular, based on the results obtained by *Claps et al.* (1993) using monthly data and confirmed by *Murrone* (1994) using daily data, parameters of the over-year groundwater can be accurately estimated only on data aggregated annually (possibly on the hydrologic year).

Based on what stated above, the first step in parameter estimation for whatever scale is to consider data aggregated on the hydrologic year. For the purpose of generality we assume that an over-year groundwater component is always present in the process, which determines the presence of correlation in the hydrologic-year series. Therefore, on these series stochastic parameters of the ARMA(1,1) model and parameters of the conceptual model, required to transfer information downward, are to be estimated.

3.1. Estimation and validation of model parameters

Conceptually-based stochastic models are not only characterised by the particular identification framework described above. Parameter estimation also takes advantage (or requires, in some cases) of the links with the conceptual model. Moreover, these links allow some hydrological parameter validations, that can be far more useful that statistical tests when the amount of data or its quality is limited. These arguments will be discusses in detail below.

3.1.1. ANNUAL SCALE

The simple structure of the stochastic model selected for annual runoff, based on the scheme of Fig. 1D, allows us to discuss some conditions in which the conceptual meaning of parameters is useful as a support to the model building steps.

The key problem is usually the limited length of the runoff series, that induces low sample autocorrelation and low significance of parameter estimates.

If observation of the series and a priori information reveal the presence of an over-year groundwater component, *Claps et al.* (1993) have suggested to legitimate the adoption of the ARMA(1,1) model even in presence of low serial lag-1 correlation or high standard error of stochastic parameters. This is supported by meaningful (and possibly validated) conceptual parameter values resulting from the ARMA parameter estimates

When data at higher resolution are available, it is also possible to further substantiate the adoption of the model by verifying the validity of corresponding models built on lower scales, since the conceptual information is preserved in the change of scale.

Equivalence between stochastic model and conceptual model plus stochastic input is not preserved in case of data transformation, usually adopted to ensure Normality and homoscedasticity in the model residual. For this reason, the modelling framework discussed here does not consider any data transformation and skewed residuals will be used for runoff generation. Linear stochastic models with Gamma marginal distribution, namely GAR and GARMA models, were actually proposed both for stationary and periodic processes to avoid transformation (*Fernandez and Salas*, 1986, 1990; *Sim*, 1987). However, this class of models lacks all the literature concerning classical ARMA model estimation and testing.

Because of the non-Normality in the model residual, the use of the Least Squares procedure was suggested (*Claps et al.*, 1993*)* to estimate parameters of the ARMA(1,1) model.

To validate conceptual parameter estimates obtained through parameters of the ARMA(1,1) model, data on lower scales are required. In particular, *Claps et al.* (1993) proposed the use of an hydrological index, called DFI (*Deep Flow Index*) and computed with monthly data, to validate the recharge coefficient c_3. This index is nothing but the average of the ratio of the annual minima of monthly runoff over the global runoff average.

If daily data are available, an additional index, related to what can be referred to as the *spring runoff*, can be used (*Claps*, 1990), which represents the relative weight of the annual minimum of daily runoff. Both these index were found (*Claps*, 1990; *Claps et al.*, 1993) strictly related to the values of the coefficient *a* estimated through the ARMA(1,1) model on the annual scale (see Tab. 2).

Table 2. *Conceptual model parameters estimated on annual runoff data for three stations in Southern Italy. Estimates are made in the hypothesis of uniform within-year input, which is reflected in the value of* r_k *(see Appendix). SFI indicates "spring" flow index.*

station	k_3	r_k	c_3	*DFI*	*SFI*
Giovenco at Pescina	2.94	0.85	0.59	0.62	0.55
Tiber at Rome	3.38	0.87	0.52	0.53	0.46
Nera at Torre Orsina	4.2	0.89	0.71	0.77	0.70

This kind of validation has a special value considering that statistical estimation of the coefficient c_3 is dependent on the estimate of the MA parameter, which is much more variable than the estimate of the other (autoregressive) parameter. Variability of the MA estimates are indicated by the high values of the related standard errors compared to these of the AR parameters (see *Claps et al.*, 1993).

3.1.2. MONTHLY SCALE

Estimation of parameters of monthly and daily runoff models requires, as said above, a preliminary estimation, on annual data, of parameters related to the over-year groundwater. This requirement is mainly due to the presence of the periodic noise represented by seasonality of the input process, that hides the structure of long-term persistence on sub-annual time series.

After that, depending on the scale of interest and on the related stochastic model, the remaining parameters are to be estimated by constraining conceptual parameters of the over-year groundwater. The estimation scheme is hierarchical, involving estimation of selected parameters step by step while increasing the data resolution.

The nature of the multiple-scale parameter estimation method is different if considering ARMA or Shot Noise models: Shot Noise parameters, in the case of response of pre-determined form (as is the sum of exponential functions) are exactly these used to define the conceptual model. Then, the procedure coincides with a constrained estimation. On the other hand, parameters of the ARMA models are stochastic and they are related to conceptual parameters in a quite complex way, particularly MA parameters. If preservation of information among scales is obtained by using conceptual parameters (as for the proposed framework), constrained estimation becomes practically impossible. For instance, suppose that parameters c_3 and k_3 of the over-year groundwater must be kept constant on the monthly scale. The stochastic model identified for monthly runoff data is a PIR-ARMA(2,2), with parameters Φ_1, Φ_2, Θ_1, Θ_2, related to conceptual parameters by means of relations (A.30-A.33). Given that, to constrain c_3 and k_3 one should relate all stochastic parameters each other in a complex way, so that it would be very difficult to determine their final values through usual methods of constrained ARMA parameter estimation.

Claps et al. (1993) suggested a way to overcome the problem related to ARMA models by means of an iterative estimation method which makes use of all the available conceptual information. The iterative method is based on the separation, in the conceptual scheme, of the over-year groundwater runoff from the total runoff. In this way, runoff is schematized as the sum of outlets of two sub-systems, one carrying the deep groundwater runoff and the other accounting for the remaining sub-annual components. Of these, only the over-year subsystem is known in his parameter values.

If one knew the input, the over-year groundwater component could be computed and separated from the rest of runoff but the input is not known in advance. To estimate the input, the PIR-ARMA(1,1) model is applied on the runoff part not coming from the over-year groundwater (see *Claps et al.*, 1993). Therefore, the iterative procedure (trial input, runoff separation, model estimation, input extraction) will converge when the estimated input is such that it produces the output of both subsystems consistently with separation of the observed runoff.

The outcome of the iterative procedure is not exactly what one expects from an ARMA parameter estimation method, since essentially what is recognised is the response of the sub-annual subsystem as an individual entity with respect to the over-year groundwater subsystem. However, conceptual meaning of stochastic parameters of the PIR-ARMA(2,2) model allows us to obtain all four stochastic parameters and the residual variance (see Appendix).

Fig. 2 shows what is achieved, in terms of runoff reconstruction, after estimation of conceptual parameters.

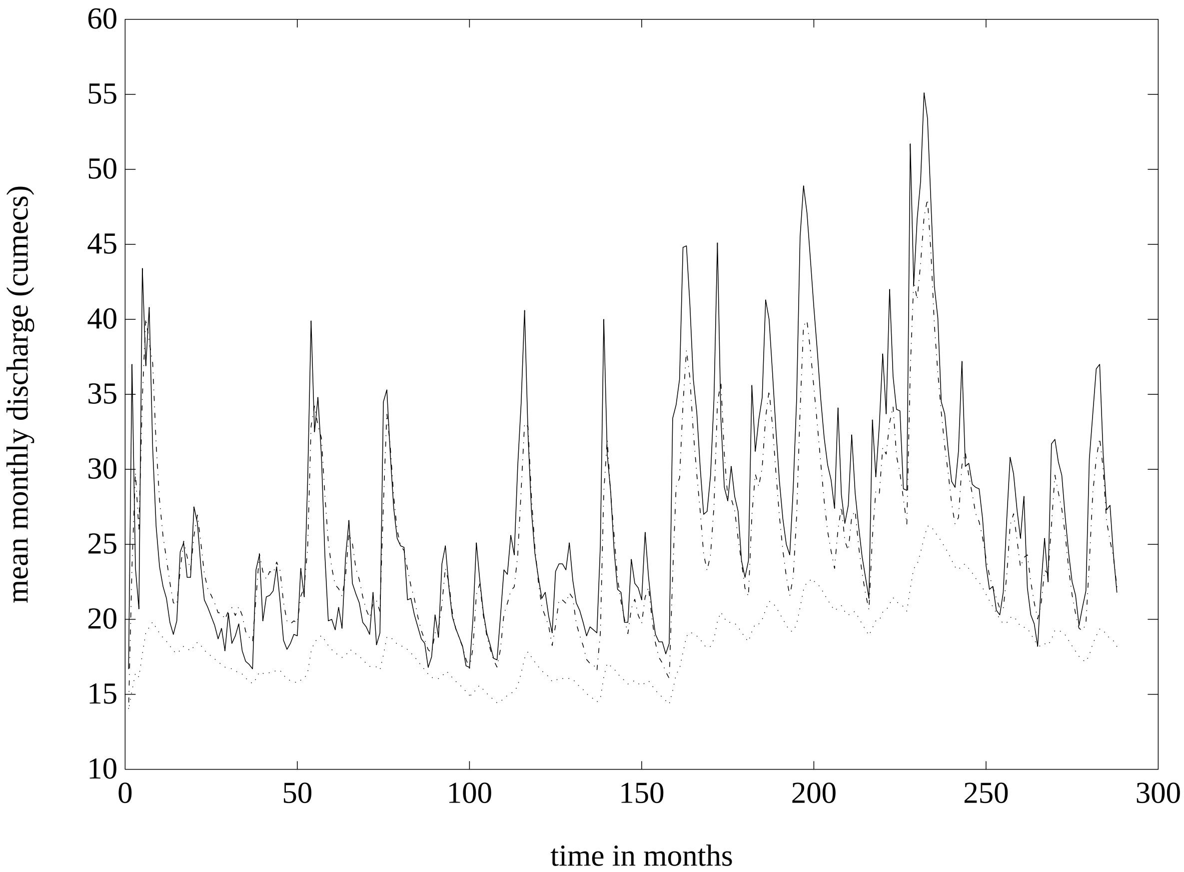

Figure 2. *River Nera. Period: October 1946 - September 1969. Observed monthly streamflow series (solid line), reconstructed over-year groundwater runoff (dotted line) and reconstructed total groundwater runoff (dash-dot line).*

The stochastic model proposed by *Claps et al.* (1993) allows estimation of the effective precipitation series as an hydrologic inverse problem. A compound probabilistic model is adopted for the net rainfall distribution. Effective rainfall evaluation and modelling will be later discussed in detail.

Conceptually-based stochastic models efficiency relies heavily in the concept of parsimony in the number of parameters, because they somewhat represent prior knowledge on the process, which is of limited complexity. What will be briefly discussed below on parameter validation will further substantiate the advantages of using constant parameters over periodic parameters models, because parsimony of conceptually-based parameters also means that there should be some possibilities of verifying the reliability of their estimates through hydrological methods.

Validation of estimates through hydrological arguments supports the usual statistical evaluation of efficiency of estimates and goes even further. In cases of lack or scarcity of data, hydrological validation methods can succeed where statistical methods fail. On the other hand, even with simple, constant parameter, models, reliable hydrological parameter validation can be performed only in limited cases, namely when the effect due to the conceptual parameter is not subject to interference by other processes. This requirement is respected in the case of the recharge coefficient of the over-year groundwater, discussed earlier, while is not respected by parameters of the sub-annual subsystem. For these latter parameters, validations can be done only indirectly, for instance looking at the appearance of the sub-annual component when plotted in the reconstructed series (Fig. 2).

3.1.3. DAILY AND T-DAY SCALE

The particular form of the Shot Noise model, where parameters of the basin response coincide with model parameters, permits immediate application of the constrained estimation, in view of the application of the scheme of parameter estimation on different scales described above. In particular, by extension of the scheme described with respect to the model of monthly runoff, parameters of the over-year and over-month groundwater components are estimated on the annual and monthly scale, respectively, and only parameters related to the interflow components are estimated with the Shot-Noise model. Examining the scheme of Fig. 1B, related to data aggregated to a suitable scale of T days, the remaining parameter c_0 results by the volume continuity condition: $\Sigma c_i = 1$.

Actual application of the Shot Noise model (*Murrone et al.*, 1992*a*) showed that better results (in terms of reproduction of data) can be obtained re-estimating on the T-day scale the recharge coefficient c_2 of the over-month groundwater. This finding confirms the statistically less reliable evaluations of the c_2 recharge coefficient (with respect to the evaluations of the k_2 storage coefficient) already experienced in the application of the ARMA models to the monthly and annual series (*Claps et al.*, 1993). The issue has been further expounded by *Murrone* (1994).

Estimation of Shot Noise parameters is carried out with an iterating Least Squares procedure (*Murrone et al.*, 1992*a*). This procedure identifies the occurrence times of the input impulses and evaluates their initial values following a technique based on a threshold runoff level (*Battaglia*, 1986), in which the threshold is set to zero. The series of inputs is forced to assume only positive values. Then, an optimisation method is carried out, based on the Nelder-Mead simplex algorithm, which minimises the sum of square deviations alternatively with respect to the series of impulses and to the set of parameters of the response. It was also possible to make an objective evaluation of the stability of parameter estimates (*Murrone et al.*, 1992*b*) using a procedure suggested by *Duan et al.* (1992), which allows to reconstruct the surface of the objective function in the subspaces defined by pairs of parameters.

Advantages of the use of a priori information on the model structure and parameters, gathered on larger scales, are evident under many points of view, starting from the convenience of limiting numerical problems associated to the contemporary estimation of all 6 parameters of expression (4).

A typical situation in which this estimation scheme is particularly efficient is the evaluation and modelling of baseflow in the analysis of daily or hourly runoff data.

Estimation of all parameters on the "local" scale does not allow one to discriminate between interflow and groundwater runoff. Consequently, models for daily data are usually unable to reproduce long periods of low flows (see *e.g. Treiber and Plate*, 1977). Moreover, even procedures for objective identification of the number of distinct components on the daily scale (*Jakeman and Hornberger*, 1993) indicate the presence of only two "active" components. This confirms that too slow components, that can actually be observed in the dry season, cannot be identified unless data are aggregated properly.

Long-term runoff must be essentially removed when quick response phenomena are considered and needs to be estimated when it represents the only contribution in the dry season. Therefore, at small scales is particularly important to correctly estimate the percentage of net rainfall entering the over-year groundwater subsystem, in order to accurately predict, for instance, runoff volumes flowing in the dry season.

Efficiency of parameter estimates, which applies essentially to the interflow subsystem, is recognised comparing observed and reproduced runoff data (Fig. 3), looking particularly at recession periods and flood peaks. Reproduction of runoff data is obtained by feeding the conceptual system with the net rainfall series identified within the estimation procedure.

3.2. Direct and inverse relationships between net rainfall and runoff

3.2.1. EFFECTIVE RAINFALL ESTIMATION AS AN HYDROLOGIC INVERSE PROBLEM

Considering runoff as the effect of the basin transformation on the effective rainfall gives the possibility of determining characters of the net rainfall process back from runoff. This possibility was exploited particularly in conceptually-based Shot-Noise models for daily runoff (see *e.g. Murrone et al.*, 1992b, for a recent review) but also in other kind of models (*e.g. Pegram*, 1980; *Hino and Hasebe*, 1981; 1984; *Vandewiele and Dom*, 1989; *Claps*, 1990; *Claps et al.*, 1993, *Wang and Vandewiele*, 1994).

Pegram (1980) set up a general framework for the basin conceptualisation, using a linear system with linear reservoirs in series and in parallel, and discussed the possibility of extracting the input process from runoff as a function of the model residual. *Hino and Hasebe* (1981) consider the daily runoff as the sum of the outlet of two or three linear systems accounting for baseflow, interflow and surface runoff. Separation of these components is obtained by low-pass digital filtering of the total runoff series, with cut-off frequencies obtained by analysis of high-order AR models applied to the total runoff. Unit hydrographs of runoff components derive from the identification of AR stochastic models on the data resulting after the separation. The net rainfall series, considered as a white noise process, is then estimated by deconvolution of the runoff components.

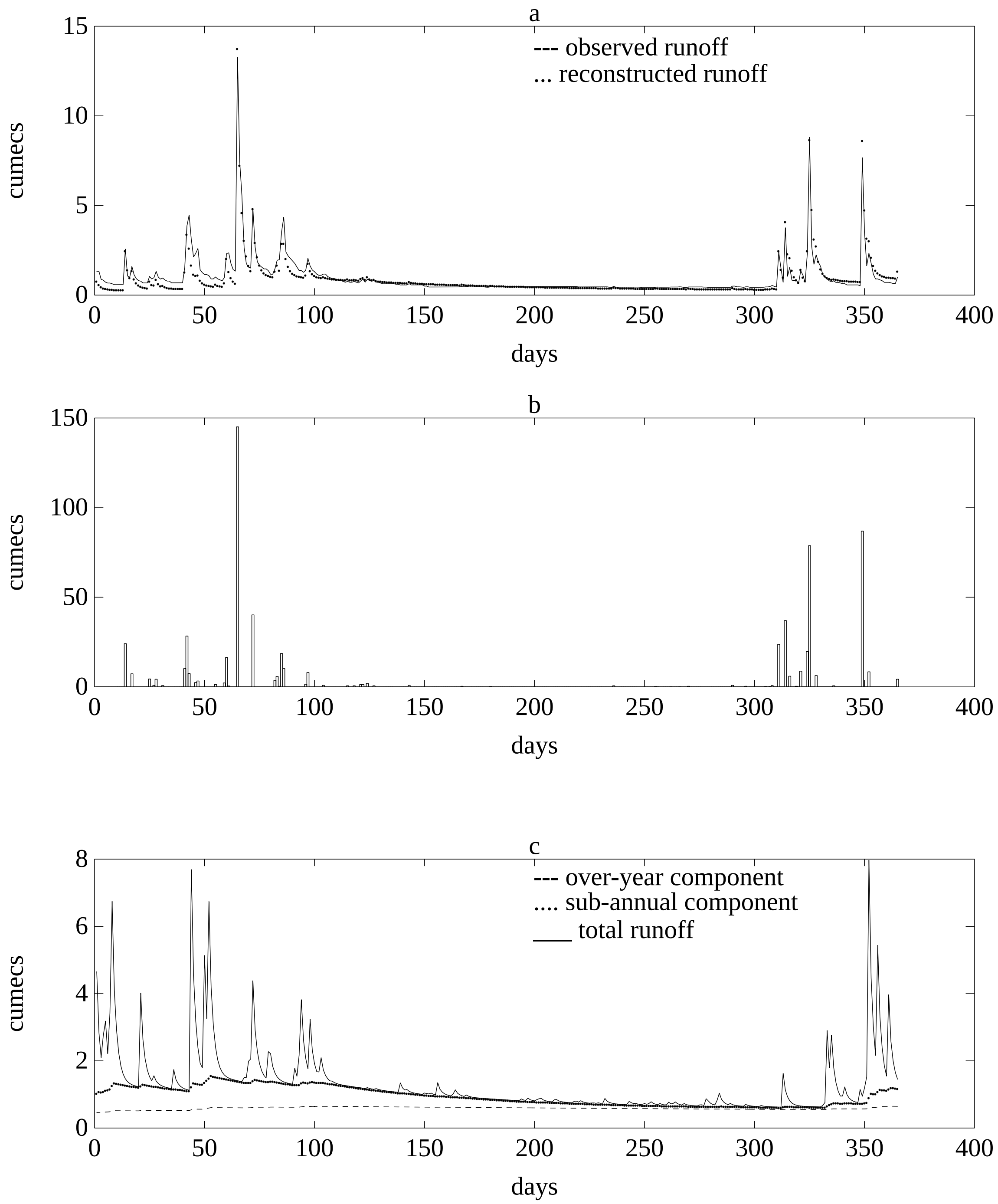

***Fig. 3**. Giovenco at Pescina, year 1962. Reconstructed vs. observed daily runoff (a), series of estimated inputs (b) and separation of components on the reconstructed series (c).*

Based on these two early approach and to evaluation of the related literature one can say that characters of the net rainfall extraction from runoff depend on the kind of model hypothesised for the basin transformations, and on the model identification and estimation techniques. If a correlation structure is hypothesised in the net rainfall, it will concur to the evaluation of the process itself. The importance of the features of runoff model building in net rainfall evaluation are evident if considering that approximations and uncertainties related to the different steps of model building concur in producing errors in the estimated input. *Todini* (1989, p. 127) expressed in general terms the dependence of errors from the approximations related to the model formulation and from all the uncertainties related to the stochastic nature of parameters.

When stochastic parameters are strictly related to a conceptual interpretation, which reduces their admissible space and guides stochastic model identification, the error term to consider when estimating net rainfall is due mainly to approximations and lack of knowledge inherent to model formulation.

Claps et al. (1993) expressed the net rainfall process as a function of the residual of the PIR-ARMA(2,2) process which contains a stochastic error term. Expressing the PIR-ARMA form as

$$d_t - \Phi_1 d_{t-1} - \Phi_2 d_{t-2} = \varepsilon'_t - \Theta_1 \varepsilon'_{t-1} - \Theta_2 \varepsilon'_{t-2} \tag{7}$$

with residual $\varepsilon'_t = \varepsilon_t + \xi_t$. ε_t is the component having conceptual meaning and ξ_t represents a Gaussian error term, with zero mean and variance σ_ξ^2. These components are considered uncorrelated and the following hold:

$$E[\varepsilon'_t] = E[\varepsilon_t]; \qquad \sigma_{\varepsilon'}^2 = \sigma_\varepsilon^2 + \sigma_\xi^2 \tag{8}$$

In this way, one can have an "estimate with error" of the net rainfall:

$$I'_t - \mu_{I'} = \frac{\varepsilon'_t}{c} \tag{9}$$

where $\mu_{I'}$ equals $E[D_t]$ and $c = (1 - c_3 r_k - c_2 r_q)$, and to the "conceptual estimate" of the net rainfall

$$I_t - \mu_I = \frac{\varepsilon_t}{c} \tag{10}$$

which, given (8), are related by

$$\mu_{I'} = \mu_I ; \qquad \sigma_{I'}^2 = \frac{\sigma_\varepsilon^2}{c^2} + \frac{\sigma_\xi^2}{c^2} = \sigma_I^2 + \frac{\sigma_\xi^2}{c^2} \tag{11}$$

Because of its meaning, the variable I_t should assume only positive values and present finite probability at zero. The probabilistic representation chosen for I_t considers the variable as the sum of a Poissonian number of events with exponentially distributed intensity (*e.g. Benjamin and Cornell*, 1970; *Öztürk*, 1981), producing the probability density function:

$$\begin{aligned} &P[I=0] = e^{-\nu} && \text{for } I=0 \\ &f_I(I) = e^{-\lambda I-\nu}\sqrt{\nu\lambda/I}\ \mathrm{I}_1[2\sqrt{\lambda I \nu}] && \text{for } I>0 \end{aligned} \tag{12}$$

where λ is the exponential parameter, ν is the Poisson parameter and $I_1(x)$ is the modified Bessel function of order 1. Moment estimators of parameters of this *Bessel* distribution are:

$$\mu_I = \nu\beta; \qquad \sigma_I^2 = 2\nu\beta^2 \qquad \gamma_I = \frac{3}{\sqrt{2\nu}} \tag{13}$$

with $\beta = 1/\lambda$ as the exponential parameter, which can be expressed in *mm*, and γ_I as the skewness coefficient.

Relations in (13) refers to the conceptual estimate I_t of the net rainfall. The probabilistic model of the net rainfall estimated with error, I'_t, is the sum of a Bessel and a Normal distributions, with parameters ν, β and $\sigma_0^2=\sigma_\xi^2/c^2$. Moments of this distribution are (*Claps*, 1992):

$$\mu_{I'} = \nu\beta; \qquad \sigma_{I'}^2 = \sigma_0^2 + 2\nu\beta^2; \qquad \gamma_{I'} = \frac{6\nu}{\left(\frac{\sigma_0^2}{\beta^2} + 2\nu\right)^{3/2}} \tag{14}$$

The above relations can be used, with some discernment (see *Claps*, 1992), to estimate parameters for the probability distribution of the effective rainfall in each season.

3.2.2. TOTAL RAINFALL TO EFFECTIVE RAINFALL TRANSFORMATION

Processes of transformation of the total rainfall to effective rainfall need to be accounted for not only if precipitation is known and is used as a causal input to the system (bivariate modelling). As seen in the previous paragraphs, considering the input as a function of the effective rainfall alone involves approximations in the evaluation of the subdivision of rainfall between direct and groundwater runoff.

Errors associated to this approximation vary depending on the degree of soil saturation and are strictly related to the total to effective rainfall transformation. On the other hand, dealing with univariate models forces one to assume simplified characters of this transformation, because the emphasis is on the estimate of the effective rainfall. Bivariate models, conversely, need to put much more detail in the rainfall-net rainfall transformation, whose characters must be more or less explicitly estimated to model the full rainfall-runoff transformation.

A main distinction can be made upon the way total to effective rainfall transformation is dealt with in hydrological modelling: transformation included in the model or left out of the model. This distinction does not necessarily coincide with that of bivariate and univariate models, since some univariate models (*e.g. Treiber and Plate*, 1977; *Hino and Hasebe*, 1981) consider nonlinearities in the rainfall-net rainfall transformation as a part of their univariate approach. The discharge level was considered by *Treiber and Plate* (1977) as a variable controlling parameters of the response of the Shot-Noise model while the digital filtering performed by *Hino and Hasebe* (1981) on runoff series produces an inverse relationship between the percentage of net rainfall entering the groundwater subsystem and the total net rainfall, resulting in a nonlinear separation law of rainfalls. Most frequently, however, univariate models are linear and cannot account for the characters of the transformation under discussion.

Bivariate models, on the other hand, must consider (implicitly or explicitly) the balance between rainfall, evapotranspiration and effective rainfall. However, even within bivariate modelling some approaches consider the net rainfall evaluation off-line with respect to the net rainfall-runoff transformation (*e.g. Weeks and Boughton*, 1987; *Jakeman and Hornberger*, 1993). It is interesting to remark that in both cases, net rainfall is obtained by digital filtering of the rainfall series, usually expressed as:

$$I_t = I_{t-1} + \frac{(P_t - I_{t-1})}{\tau} \qquad (15)$$

with τ as the time constant of the filter.

Unlike the majority of bivariate models (see *e.g. Franchini and Pacciani*, 1991; *Chiew et al.*, 1993), that are deterministic, the approaches by *Weeks and Boughton* (1987) and *Jakeman and Hornberger* (1993), taken among others, establish a stochastic correspondence between rainfall and runoff. In particular, the net rainfall-runoff model is linear and stochastic and is of the ARMA type. More precisely, in the ARMA representation the net rainfall process is considered stochastic (*i.e.* containing the error term) while if net rainfall is considered as a deterministic variable and the error is an additive term representing the residual an ARMAX model (ARMA with eXogenous variable) is obtained (*Spolia and Chander*, 1974).

The main problem with all traditional (deterministic and stochastic) bivariate approaches is that, as recognised by *Franchini and Pacciani* (1991), the water balance within the soil is completely unobserved. This frustrates efforts done toward a detailed global schematization of the process and attributes empirical nature even to the separate modelling of the rainfall-net rainfall transformation.

This makes not advantageous to undertake a detailed modelling of all subprocesses acting within the soil, unless separate evaluation of the net rainfall process is available.

3.2.3. TOWARD AN INTEGRATED SCHEME WITH NONLINEAR RAINFALL - NET RAINFALL TRANSFORMATION MODULE

A basic distinction between bivariate and univariate models can be made with reference to the way the rainfall-net rainfall transformation is considered. Bivariate models operate a simultaneous reproduction of both runoff-net rainfall and net rainfall-rainfall transformation while univariate models take only the former into account.

When facing problems of short-term forecasting and precipitation data are available, a new approach could be used, that maximises the information represented by runoff data. Univariate, conceptually-based stochastic models can be used to provide an estimate of the net rainfall while a bivariate nonlinear rainfall-net rainfall model can account for the transformations occurring on total precipitation.

Errors related to the constancy of parameters can be explicitly accounted for in the rainfall - net rainfall module, where most of the nonlinearities of the physical process are concentrated. However, in the author's opinion, conceptually-based linear univariate models for net rainfall evaluations are preferable to nonlinear models because parameter estimates can be better determined and validated.

4. FINAL REMARKS

Conceptually-based stochastic modelling of streamflow series fills, to a large extent, the requirements connected nowadays to the use of the streamflow synthesis. As underlined by *Yevjevich* (1991), among others, it is meaningful to persist in building models only if the emphasis is on the understanding of the physical process (*structural analysis* of the process). Moreover, since the statistical tools have been built and tested, it becomes increasingly important to focus on *what* to model rather than on *how* to model. In other terms, the quality of data must be taken into account when building or using a model.

The importance of the above two points is the key of the interest of the conceptual approach to runoff modelling, that can be synthesised in: *physical consistency* and *overall efficiency*. This second characteristic means: objective criteria for model identification, simple stochastic model structure and conceptually-related parameters for efficient parameter estimation and possibility of their validation.

In this paper, most of the emphasis is put on the different aspects concerning the *efficiency* of conceptually-based models. It is thought that this concept is the one that allow a correct evaluation of competing physically-consistent approaches to runoff time series (*e.g.* linear - nonlinear, univariate - bivariate) and that most of the work needed in the field under discussion should tend to the selection of the *most efficient* model given the physical information and the amount and quality of data.

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APPENDIX: EQUIVALENCE IN REPRESENTATION BETWEEN LINEAR SYSTEMS AND ARMA STOCHASTIC MODELS

1. Linear reservoir discretization

The continuous-time response of a linear reservoir to an input r(τ) is (*e.g. Chow et al.*, 1988):

$$Q(t) = Q_0\, e^{-t/k} + \int_0^t \frac{1}{k}\, e^{-(t-\tau)/k}\, r(\tau)\, d\tau \tag{A.1}$$

where Q is the discharge, k is the reservoir *storage coefficient* and $Q_0=Q(t=0)$. Using an impulse-response formulation, equation (A.1) can be written as

$$Q(t) = \int_{-\infty}^{t} r(\tau)\, h(t-\tau) d\tau \tag{A.2}$$

with $r(\tau)$ as the input and $h(t-\tau)=1/k\, e^{-(t-\tau)/k}$ as the impulse response of the linear reservoir.

Integration of (A.1) over a unit-time interval gives

$$D_1 = \int_0^1 Q(t) = \int_0^1 Q_0 e^{-t/k}\, dt + \int_0^1 \frac{1}{k} e^{-t/k} \int_0^t e^{\tau/k}\, r(\tau)\, d\tau\, dt \tag{A.3}$$

where D_1 is the outflow volume. Given the initial volume $V_0=kQ_0$, by definition of linear reservoir, equation (A.3) becomes

$$D_1 = (1-e^{-1/k})V_0 + \int_0^1 \frac{1}{k} e^{-t/k} \int_0^t e^{\tau/k}\, r(\tau)\, d\tau\, dt \tag{A.4}$$

To discretize the above equation, the form of the input function is needed. A generalised discretized form can be obtained by introducing the *recharge recession coefficient*

$$r_k = 1 - \int_0^1 \frac{1}{k}\, e^{-t/k} \int_0^t \frac{1}{R}\, e^{\tau/k}\, r(\tau)\, d\tau\, dt \tag{A.5}$$

where $R = \int_0^1 r(\tau)\, d\tau$ is the *recharge* volume within the interval. Assuming, for the generic $(t-1,t)$ unit-time interval, D_t as the runoff, $V_{t-1} = V_0$ as the storage at the beginning of the interval, R_t as the recharge in the interval and $v_k=e^{-1/k}$, (A.4) can be rewritten as

$$D_t = (1-v_k)\, V_{t-1} + (1-r_k)\, R_t \tag{A.6}$$

Calling v_k the *recession coefficient*, from equation (A.6) emerges clearly that coefficient r_k is equivalent to v_k but responsible for the recession of the within-interval recharge volume. Both r_k and

v_k can only assume values between 0 and 1. The larger r_k, the smaller the amount of recharge R produced as runoff. Analysis of (A.5) shows that r_k depends on the form of the input function and on the value of k. For instance, r_k increases with increasing k and with the uniformity of the input. If the input is not continuous r_k increases as the input concentrates at the end of the interval. It is particularly important to correctly estimate r_k when a deterministic within-period variability of the input can be recognized, as in the case of series aggregated annually, where the input displays a seasonal pattern.

In case of uniform recharge $r(\tau)=R/\Delta$ over a sub-interval of duration $\Delta \le 1$ and initial time T_i , the expression of r_k is

$$r_k = \frac{k}{\Delta}\,(1-e^{-\Delta/k})\;[1 - (1 - e^{(1-T_i-\Delta)/k})] \tag{A.7}$$

that reduces to $r_k = e^{-(1-T)/k}$ for $\Delta=0$. For $T_i=0$ and $\Delta=1$ relation (A.7) becomes

$$r_k = k\,(1 - e^{-1/k}) \tag{A.8}$$

For $\Delta=0$ and $T=0$, which corresponds to impulse recharge at the beginning of the interval, (A.7) gives $r_k = e^{-1/k} = v_k$. When there is no particular evidence of a within-period rainfall pattern, as in the case of monthly data, relation (A.8) for uniform $r(\tau)$ is chosen.

A seasonal pattern in hydrologic data can be estimated, with good accuracy, by means of Fourier series. In this case, one can use the generic periodic function $f(t)$ as the dimensionless input $r(t)/R$:

$$f(t) = A_0 + \sum_{n=1}^{N} A_n \cos\left(\frac{2n\pi}{T}t + \Phi_n\right) \tag{A.9}$$

with N = number of harmonics and A_0 = mean of $f(t)$ over the period T. For fixed N, the above expression has $2N$+1 parameters, namely A_0, $\{A_1, A_2, ...A_N\}$ and $\{\Phi_1, \Phi_2, ...\Phi_N\}$, that can be estimated, for instance, by means of a least squares method. However, given the forms assumed by the monthly rainfall means curves, two harmonics can be considered adequate for an accurate fitting.

Assuming $A_0=1$, substitution of (A.9) into (A.5), with the interval T set to 1 year, produces (*Claps and Murrone*, 1993):

$$\begin{aligned} r_k = 1-(ke^{-1/k}+1-k) - \sum_{n=1}^{N} \frac{A_n k e^{-1/k}\left[\cos\Phi_n + 2\pi nk\ \sin\Phi_n\right]}{4(\pi kn)^2+1} + \\ + \sum_{n=1}^{N} \frac{A_n\left[2\pi kn\cos(2\pi n+\Phi_n) - \sin(2\pi n+\Phi_n) + (4(\pi kn)^2+1)\sin\Phi_n\right]}{2\pi n(4(\pi kn)^2+1)} \end{aligned} \tag{A.10}$$

or, with a different arrangement of terms,

$$(-1 + e^{1/k})\, k \sum_{n=1}^{N} \frac{\left[1 + 4\,(kn\pi)^2 - A_n \cos(\Phi_n) + 2\,A_n\, kn\pi \sin(\Phi_n)\right]}{e^{1/k}\left[1 + 4\,(kn\pi)^2\right]} \tag{A.11}$$

2. equivalent stochastic models of different conceptual schemes

SINGLE LINEAR RESERVOIR

The outlet D_t of a single linear reservoir in a unit-time interval $(t-1,t)$ is expressed by (A.6) as a function of the initial storage V_{t-1} and of the input R_t (recharge) in the interval. The mass-balance equation for the groundwater storage is

$$V_t = v_k V_{t-1} + r_k R_t \tag{A.12}$$

Eliminating V_{t-1} between (A.12) and (A.6), one obtains

$$V_t = \frac{v_k D_t}{1-v_k} - \frac{v_k (1-r_k) R_t}{1-v_k} + r_k R_t \tag{A.13}$$

This relation can be rewritten using V_{t-1}, D_{t-1} and R_{t-1} instead of V_t, D_t and R_t, without loss of generality. Then, V_{t-1} can be eliminated again, using (A.6), producing

$$\frac{D_t}{1-v_k} - \frac{(1-r_k)\, R_t}{1-v_k} = \frac{v_k D_{t-1}}{1-v_k} - \frac{v_k (1-r_k) R_{t-1}}{1-v_k} + r_k R_{t-1} \tag{A.14}$$

and, rearranging,

$$D_t - v_k D_{t-1} = (1 - r_k)\, R_t - (v_k - r_k)\, R_{t-1} \tag{A.15}$$

If R_t is an independent stochastic process, (A.15) is the expression of an ARMA(1,1) (Autoregressive and Moving Average of order one and one) process (*Box and Jenkins*, 1970). In Box-Jenkins notation, this process is represented as

$$d_t - \Phi\, d_{t-1} = \varepsilon_t - \Theta\, \varepsilon_{t-1} \tag{A.16}$$

with $d_t = D_t - E[D_t]$ and $\varepsilon_t = (1-r_k)(R_t - E[R_t])$ as zero-mean variables. Coefficients $\Phi = v_k$ and $\Theta = (v_k - r_k)/(1-r_k)$ are the autoregressive and moving average parameters, respectively.

If the within-period distribution of the recharge is pre-determined, coefficients Φ and Θ of (A.16) are not independent. As seen before, In the limit case of impulse recharge at the beginning of the interval, r_k equals v_k and (A.15) simplifies in

$$D_t - v_k D_{t-1} = (1 - v_k)\, R_t \tag{A.17}$$

which is equivalent to an AR(1) process.

On a seasonal basis, if the storage constant is considered as varying with the seasons, (A.17) becomes a PAR(1) process, as shown by *Salas and Obeysekera* (1992) with reference to a similar conceptual scheme, that can be written as

$$d_{n,m} - \Phi_m \, d_{n,m-1} = \varepsilon_{n,m} \tag{A.18}$$

in which n denotes the year and m denotes the season. Coefficients Φ_m relate to the seasonal $v_k(m)$ as (*Salas and Obeysekera*, 1992)

$$\Phi_m = v_k(m-1)\,[1-v_k(m)] \,/\, [1-v_k(m-1)] \tag{A.19}$$

where $v_k(m) = e^{-1/k(m)}$.

LINEAR RESERVOIR PLUS DIVERSION.

The simplest variant to the basic system just discussed arises by considering a diversion added to the linear reservoir. In the Thomas-Fiering model of watershed (see *Fiering*, 1967), for instance, a diversion in parallel to the linear reservoir accounts for the part of the input that does not recharge the deep groundwater and reaches the basin outlet within the end of the time interval considered (direct runoff).

With reference to Figure 1d, the groundwater recharge R_t in the interval t is the fraction $c_3 I_t$ of the effective rainfall I_t and the direct runoff is $(1-c_3)I_t$. Including the direct runoff into the mass balance equations (A.6) and (A.1) and rearranging, an ARMA(1,1) process is obtained (*Salas et al.*, 1981):

$$d_t - \Phi \, d_{t-1} = \varepsilon_t - \varepsilon_{t-1} \tag{A.20}$$

Considering the effective rainfall as input allows to obtain explicit relations between conceptual and stochastic parameters:

$$\Phi = v_{k3} = e^{-1/k_3}; \quad \Theta = \frac{v_{k3} - c_3 r_{k3}}{1 - c_3 r_{k3}} \tag{A.21}$$

$$c_3 = \frac{\Phi - \Theta}{r_k(1-\Theta)}; \qquad k_3 = \frac{1}{\ln \Phi}; \tag{A.22}$$

and to link the zero-mean effective rainfall $i_t = I_t - E[I_t]$ to the stochastic model residual ε_t:

$$\varepsilon_t = (1 - c_3 r_k) i_t; \quad i_t = \frac{\varepsilon_t}{1 - c_3 r_k} \tag{A.23}$$

If σ_ε^2 is the residual variance, the variance of the net rainfall process is

$$\sigma_I^2 = \frac{\sigma_\varepsilon^2}{(1-c_3 r_k)^2} \tag{A.24}$$

The obvious constraints $0 \le c_3 \le 1$ and $k_3 \ge 0$ restrict the parameter space of the process (A.20) (*Salas et al.*, 1981), with respect to that of a general ARMA(1,1) process. Hence, process (A.20) must be considered as a *restricted* ARMA process.

With reference to seasonal runoff, *Salas and Obeysekera* (1992) showed that, considering parameters c_3 and k_3 as varying with the seasons, the scheme depicted in Figure 1d leads to a PARMA(1,1) process. If only one parameter is assumed nonseasonal, a PARMA(1,1) process with nonseasonal AR or MA parameter is obtained. In all cases, retaining the results from the mentioned authors, explicit relationships similar to those given in (A.21) and (A.22) can be obtained between $c_{3,m}$, $k_{3,m}$ and Φ_m , Θ_m.

TWO PARALLEL LINEAR RESERVOIRS PLUS DIVERSION.

This case is represented in Figure 1c where, as immediate extension of the previous scheme, c_2, k_2, W and r_{k2} have the same meaning of c_3, k_3, V and r_{k3} and, accordingly, $v_{k2}=e^{-1/k2}$. For this conceptual system, runoff D_t is given by

$$D_t = (1-v_{k3})V_{t-1} + (1-v_{k2})W_{t-1} + c_3(1-\mathrm{r}_{k3})I_t + c_2(1-\mathrm{r}_{k2})I_t + (1-c_3-c_2)\, I_t \tag{A.25}$$

The volume balance equations for the groundwater storage are:

$$V_t = v_{k3}\, V_{t-1} + c_3\, r_{k3}\, I_t \tag{A.26}$$

$$W_t = v_{k2}\, W_{t-1} + c_2\, \mathrm{r}_{k2}\, I_t \tag{A.27}$$

Putting in (A.25) the expressions of W_{t-1} and V_{t-1} obtained from equations (A.26) and (A.27) and rearranging, gives one equation in D_t, D_{t-1}, D_{t-2}, I_t, I_{t-1}, I_{t-2}:

$$\begin{aligned} & D_t - (v_{k3} + v_{k2})D_{t-1} + (v_{k3}v_{k2})D_{t-2} = \\ & = (1 - c_3\, r_{k3} - c_2\, r_{k2})I_t - [v_{k3} + v_{k2} - c_3\, r_{k3}(1+v_{k2}) - c_2\, r_{k2}(1+v_{k3})]I_{t-1} + \\ & - (c_3\, r_{k3}v_{k2} + c_2\, r_{k2}v_{k3} - v_{k3}v_{k2})\, I_{t-2} \end{aligned} \tag{A.28}$$

With reference to the zero-mean variables d_t and i_t, if $\varepsilon_t = (1 - c_2\mathrm{r}_{k2} - c_3\mathrm{r}_{k3})\mathrm{i}_\mathrm{t}$ is an independent stochastic process, the above representation is equivalent to an ARMA(2,2) process

$$d_t - \Phi_1\, d_{t-1} - \Phi_2\, d_{t-2} = \varepsilon_t - \Theta_1\, \varepsilon_{t-1} - \Theta_2\, \varepsilon_{t-2} \tag{A.29}$$

Following the reasoning by *Salas and Obeysekera* (1992) and letting c_2, c_3, k_2 and k_3 vary with the seasons, a PARMA(2,2) process arises.

Relations between conceptual and stochastic parameters for the ARMA(2,2) are:

$$\Phi_1 = v_{k3} + v_{k2} \tag{A.30}$$

$$\Phi_2 = -v_{k3}v_{k2} \tag{A.31}$$

$$\Theta_1 = \frac{v_{k3} + v_{k2} - c_3 r_{k3}(1+c_2) - c_2 r_{k2}(1+c_3)}{1 - c_2 r_{k2} - c_3 r_{k3}} \tag{A.32}$$

$$\Theta_2 = \frac{c_3 r_{k3} v_{k2} + c_2 r_{k2} v_{k3} - v_{k3} v_{k2}}{1 - c_2 r_{k2} - c_3 r_{k3}} \tag{A.33}$$

$$v_{k3} = \frac{\Phi_1 + \sqrt{(\Phi_1^2 + 4\Phi_2)}}{2} \tag{A.34}$$

$$v_{k2} = \frac{\Phi_1 - \sqrt{(\Phi_1^2 + 4\Phi_2)}}{2} \tag{A.35}$$

$$c_3 = \frac{M - N - M\, c_2 r_{k2}}{M\, r_{k3}} \tag{A.36}$$

$$c_2 = \frac{-(\Theta_1 - \Theta_2)N + (\Phi_1 - \Phi_2)M + (1+2v_{k2})(N-M)}{2\, M\, (v_{k3} - v_{k2}) r_{k2}} \tag{A.37}$$

where $N = (1 - \Phi_1 - \Phi_2)$ and $M = (1 - \Theta_1 - \Theta_2)$. Relations (A.30)-(A.33) or (A.34)-(A.37) ensure equivalence of the means of D_t and I_t (*Claps*, 1990). The net rainfall variance can be obtained from the residual variance σ_ε^2 through the relation

$$\sigma_I^2 = \frac{\sigma_\varepsilon^2}{(1 - c_2 r_{k2} - c_3 r_{k3})^2} \tag{A.38}$$

In the general ARMA(2,2) model, stationarity and invertibility conditions provide admissible regions for the AR and MA parameters. Limiting the analysis to the autoregressive parameters Φ_1 and Φ_2, a triangular admissible region results (*Box and Jenkins*, 1970, par. 3.2.4.) from the stationarity conditions $-1 < \Phi_2 < 1$; $\Phi_2 - \Phi_1 < 1$; $\Phi_2 + \Phi_1 < 1$. Given that $0 < v_{k2} < 1$ and $0 < v_{k3} < 1$, equations (A.34)-(A.35) provide conceptual constraints for Φ_1 and Φ_2 :

$$\Phi_1^2 + 4\,\Phi_2 > 0 ; \qquad \Phi_2 < 0 ; \qquad \Phi_1 > 0 \tag{A.39}$$

which restrict the admissible space of the two parameters to a quite small region. Again, the conceptual representation determines a restricted ARMA process. It is interesting to remark that if one of the storage coefficients, for instance k, is large, v_{k3} approaches unity and this leads to $\Phi_1 + \Phi_2 \approx 1$, which is close to the non-stationarity condition $\Phi_1 + \Phi_2 = 1$. In other words, the outlet of a system with a very slow response (such as a reservoir with a recession coefficient $v_{k3} \approx 1$) may be confused with a trend.