REGIONAL FLOOD FREQUENCY ANALYSIS WITH A THEORETICALLY DERIVED DISTRIBUTION FUNCTION

P. CLAPS [1], M. FIORENTINO [2] and V. IACOBELLIS [3]
[1] Dipartimento di Idraulica, Trasporti e Infrastrutture Civili - Politecnico di Torino
claps@polito.it, C.so Duca degli Abruzzi 24, 10129 Torino, Italy
[2] Dipartimento di Ingegneria e Fisica dell'Ambiente - Università della Basilicata
fiorentino@unibas.it, Contrada Macchia Romana, 85100, Italy
[3] Dipartimento di Ingegneria Civile e Ambientale - Politecnico di Bari
v.iacobellis@poliba.it, Via E. Orabona, 4, 70125, Bari, Italy

ABSTRACT

The estimation of flood frequency curves in ungauged basins is an important field of application of advanced research concerning the knowledge on physical processes and the development of statistical tools. Regional statistical analysis and physically-consistent derivation of probability density functions (*pdf*) are the key fields to rely on for providing robustness to estimation of flood quantiles, and for transferring hydrological information between basins. In this context, physically consistent reasoning applied to the regional statistical analysis can be closely connected with some basis of a geomorphoclimatic approach for derivation of the flood *pdf*.

In this paper, perceptible effects of climatic and physiographic basin parameters on the flood *pdf* were sought. In particular, estimates of flood distribution parameters were related with other physical features of the investigated basins. Results obtained on data from the Calabria region (Southern Italy) were compared with those previously achieved with regard to conterminous regions. The relationships between climate and flood frequency distribution seem to confirm the potential of the link between geomorphoclimatic and statistical parameters for use in regional analyses. Finally, issues related to the scaling of flood distribution parameters with basin area are investigated and commented.

KEYWORDS: Floods, Climate, Hydrologic losses.

1. Introduction

Existing analyses of rainfall and flood annual maxima within a large region of Southern Italy are revised in light of the theoretical model for derivation of flood frequency distribution proposed by *Iacobellis and Fiorentino* (2000). The analysis was performed with the main purpose of understanding the hydrological processes underlying the flood generation process. The model applicability over a wide range of natural basins, different for climate, vegetation coverage, soil structure and permeability, was also assessed. In particular, the analysis previously performed over basins belonging to the Italian regions of Puglia and Basilicata (*Fiorentino and Iacobellis*, 2001) was extended to Southwest Italy (peninsula of Calabria).

The overall procedure is composed of three different steps. First, standard regional methods are applied to the observed series of annual maxima of both rainfall and flood records. The estimated parameters are exploited in the second step to obtain some of the unknown parameters of the theoretical distribution. Being physically based, the latter allow one to identify some components of the rainfall runoff mechanisms playing as sub-models of the theoretical distribution. In the last step, heterogeneities and homogeneities are analyzed and compared in the light of results obtained in conterminous regions. Finally, issues such as variability of moment distribution in time and space are discussed.

2. Theoretical framework

The quoted theoretical model for analytical derivation of the flood frequency distribution is based on a simple rainfall-runoff model in which the peak flow Q_P is treated as a stochastic variable function of two other mutually dependent stochastic variables, namely the mean areal rainfall intensity $i_{a,\tau}$ and the peak runoff contributing area a:

$$(1)\quad Q_P = x\ (i_{a,\tau} - f_a)\, a + q_o$$

where x is a constant routing factor, $i_{a,\tau}$ is the areal rainfall intensity in the duration equal to the lag-time τ_a within the source area a, f_a is the average water loss rate, within the same duration τ_a and area a, q_o is a constant base flow.

The time-space behavior of the involved quantities is basically controlled by the commonly observed geomorphologic power-type relationship between basin lag-time τ_A and basin area A which can be written as:

$$(2)\quad \tau_a = \tau_1\, a^{\nu} \quad \text{with} \quad \tau_1 = \tau_A A^{-\nu}$$

where τ_A is the lag-time of the basin and ν usually assumes values close to 0.5. The mean areal rainfall intensity $E[i_{a,\tau}]$ is usually found to scale with a according to the power law

$$(3)\quad E\left[i_{a,\tau}\right] = i_1\, a^{-\varepsilon} \quad \text{with} \quad i_1 = E[i_A]A^{\varepsilon}$$

where i_1 is rainfall intensity referred to the unit area. Also, in the model f_A is in general supposed to scale with the basin area A through a relationship of the type:

$$(4)\quad f_a = f_1\, a^{-\varepsilon'} \quad \text{with} \quad f_1 = f_A\, A^{\varepsilon'}$$

in which f_A represents the average water loss rate when the entire basin contributes to the flood peak. Indeed, τ_A, $E[i_A]$, f_A, ν, ε and ε' are characteristic features of basins.

Under the assumption of a rainfall process with poissonian occurrences and Weibull distributed intensity, the average areal water loss f_A may be related to the ratio between the average annual rates of rainfall and flood events, respectively Λ_p and Λ_q, by means of the equation (*Fiorentino and Iacobellis*, 2001):

$$(5)\quad f_A = \frac{E[i_A]}{\Gamma(1+1/k)}\left[\log\left(\frac{\Lambda_p}{\Lambda_q}\right)\right]^{1/k}$$

where k is the exponent of the Weibull distribution of rainfall intensity and $\Gamma(\cdot)$ is the gamma function.

Given a series of rainfall annual maxima, parameters p_1 and n of the at-site intensity-duration-frequency curve (*idf*) are derived according to equation:

$$(6)\quad E[p_t\ ] = p_1 t^{n-1}.$$

This is referred to the expected value of the annual maximum rainfall intensity p_t, in the duration t.

Assuming the hypotheses of Weibull distribution of rainfall intensity and poissonian occurrence of events, the distribution of annual maxima turns out to be a Power Extreme Value (PEV) type.

In the same framework, exploiting the relationship between the averages of *annual maxima* and of the *base process*, the average areal rainfall intensity may be estimated by means of the equation:

$$(7)\quad E[i_A] = \frac{p_1 \tau_A^{n-1}\left[1 - \exp\left(-1.1\tau_A^{0.25}\right) + \exp\left(-1.1\tau_A^{0.25} - 0.004A\right)\right]}{\Lambda_p \sum_{j=0}^{\infty} \frac{(-1)^j \Lambda_p{}^j}{j!(j+1)^{(1/k+1)}}}.$$

This is valid for PEV distributed data, where p_1 is the mean annual rainfall depth in 1 hour. Equation (7) includes the well-known Weather Bureau areal reduction formula, here used replacing time with the basin lag-time τ_A.

3. Model Application

In *Fiorentino and Iacobellis* (2001) the model above was applied to 20 annual flood series in Puglia and Basilicata, which are contiguous (North and North-East) to Calabria (Figure 1). In this paper we applied the model to data from the latter region. This region presents substantial differences as regarding climate and basin permeability, which somehow integrate the conditions found in the previous applications.

Annual flood series of 13 sites in Calabria (figure 1) with record length ranging between 15 and 49 years were analyzed. Their main features are shown in table 1.

n.	Station	A	$\mathbf{t}_A$	q_o	Zone	L_p	p_1	n	I
		Km^2	h	m^3/s			mm/h		
1	Crati at Conca	1339	5.5	52.04	Central	20	24.3	0.40	0.61
2	Esaro at La Musica	520	4.7	22.36	Central	20	22.1	0.46	0.77
3	Coscile at Camerata	285	3.7	8.39	Central	20	22.3	0.55	0.65
4	Trionto at Difesa	32	2.8	1.17	Central	20	31.0	0.50	0.90
5	Tacina at Rivioto	79	3	3.40	Jonian	10	32.7	0.59	1.43
6	Alli at Orso	46	3	2.34	Central	20	33.2	0.52	1.26
7	Melito at Olivella	41	3	1.75	Central	20	33.2	0.47	0.72
8	Corace at Grascio	182	3.8	8.84	Central	20	29.8	0.45	0.90
9	Ancinale at Razzona	116	3.9	7.12	Jonian	10	37.9	0.54	1.34
10	Alaco at Mammone	15	1.3	0.96	Jonian	10	39.6	0.63	1.66
11	Amato at Marino	113	4.6	5.32	Central	20	28.8	0.43	0.86
12	Lao at Piè di Borgo	280	3.7	12.43	Thyrrenian	34	27.8	0.46	1.16
13	Noce at La Calda	42.5	1.3	2.72	Thyrrenian	34	26.6	0.47	1.58

***Table 1.** Investigated basins and their main hydrologic features.*

Basin areas A and lag-times τ_A were taken from *Versace et al.* (1989). These values were found to conform to the scaling law in equation (2). In particular, assuming $\nu = 0.50$ we estimated for basins #1, #2, #3, #12 and #13 the value $\tau_1 = 0.18$ [$h{\cdot}km^{-2\nu}$] very close to the one observed in Basilicata, which was $\tau_1 = 0.19$ [$h{\cdot}km^{-2\nu}$]. For the other basins we found $\tau_1 = 0.36$ [$h{\cdot}km^{-2\nu}$].

The climatic classification was performed by means of the climatic index (Thornthwaite, 1948):

$$(8)\quad I = \frac{h - E_p}{E_p}.$$

In the equation, h is the mean annual rainfall depth and E_p the mean annual potential evapotranspiration calculated according to Turc's formula (*Turc*, 1961), dependent on the mean annual temperature only.

The rainfall annual maxima in 225 raingauge stations (with record length not less than 20 years) were used in the relevant regional statistical analysis. In this analysis, the key parameter to estimate is the mean annual number, Λ_p, of independent rainfall events. Local and regional values of Λ_p were obtained by means of a regional model based on a Two Component Extreme Value distribution (*Rossi et al*., 1984). Parameters Λ_1, θ_1, and Λ_2, θ_2 of the TCEV were estimated using a Maximum Likelihood (TCEV-ML) procedure (*Gabriele and Iiritano*, 1994) with hierarchical estimation of parameters (*Fiorentino et al*., 1987), based on the homogeneous areas found in *Versace et al.* (1989). Figure 1 shows the homogeneous zones, while the estimated values of Λ_p, which are related to the coefficient of variation of the rainfall annual maxima, are displayed in table 1.

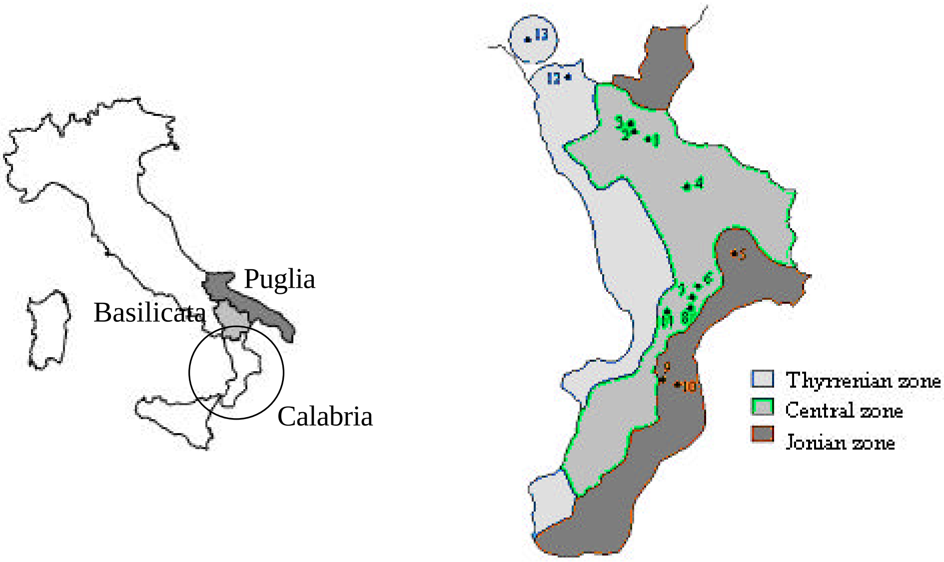

Figure 1. *Calabria region: gauging stations considered and rainfall homogeneous zones.*

A regional estimation based on a PEV-ML procedure (*Villani*, 1993) was then applied to the same 225 stations, providing an estimate of the shape factor $k = 0.53$. The estimates of $E[i_A]$ obtained from equation (7) are shown in table 2 and Figure 2.

Basins belonging to the Thyrrenian and Central zones follow the power relationships in equation (3) with parameters $i_1 = 11.5$ [$mm\ h^{-1}\ km^{-2\varepsilon}$] and $\varepsilon = 0.28$ ($R^2 = 0.94$), while basins of the Jonian Zone (#5, #9 and #10) are characterized by higher values, that seem to follow equation (3) with parameters: $i_1 = 28.8$ [$mm\ h^{-1}\ km^{-2\varepsilon}$], $\varepsilon = 0.32$ ($R^2 = 0.98$). The slopes obtained are not far from the one observed in Basilicata ($i_1 = 0.33$).

In order to obtain estimates of the mean annual number Λ_q of floods shown in table 2, the recorded series of annual flood maxima were analyzed by a regional GEV-PWM procedure (*Hosking and Wallis,* 1993). In particular, the Λ_q values were obtained using the regional estimate of L-skewness and the at-site estimates of the L-coefficient of variation. In other words, we assumed homogeneity all over the region even if we should acknowledge that some of the basins (namely, #2, #4, #11 and #13) resulted of uncertain homogeneity with those of Puglia and Basilicata. In the analysis we excluded basin #1 whose coefficient of variation, according to *Versace et al.* (1989), is affected by the presence of reservoirs.

For the investigated basins in Calabria, the behavior of the ratio Λ_q / Λ_p as a function of the basin area is shown in Figure 3. One can note a significant trend of this ratio to decrease as A increases, which means that rainfall events tend to reduce their capability to yield floods as the basin area becomes larger. As will be detailed in the next section, this behavior is mainly due to the scaling properties shown by f_A with the area A. In figure 4, f_A estimates, achieved by equation (5), are shown.

These results are commented hereafter in comparison with what was observed in other regions nearby.

n.	Station	$E[i_A]$	L_q	L_q / L_p	f_A
1	Crati at Conca	1.6	7.5	0.37	1.4
2	Esaro at La Musica	1.8	3.0	0.15	3.5
3	Coscile at Camerata	2.4	3.2	0.16	4.5
4	Trionto at Difesa	4.1	10.7	0.54	1.0
5	Tacina at Rivioto	6.7	4.0	0.40	3.2
6	Alli at Orso	4.3	4.0	0.20	5.8
7	Melito at Olivella	4.1	4.8	0.24	4.5
8	Corace at Grascio	2.9	4.5	0.22	3.4
9	Ancinale at Razzona	6.4	3.3	0.33	4.4
10	Alaco at Mammone	12.1	3.5	0.35	7.4
11	Amato at Marino	2.5	5.0	0.25	2.6
12	Lao at Piè di Borgo	2.2	5.5	0.16	3.9
13	Noce at La Calda	4.2	13.7	0.40	2.9

Table 2. *Rainfall and water loss parameter estimates.*

4. Comparisons and concluding remarks

Analogous analyses were developed with regard to Basilicata and Puglia regions. As regards the relationship Λ_q / Λ_p - basin area, results are shown in figure 3 where basins with negative or slightly positive climatic index in Puglia and Basilicata are included. Instead, results regarding f_A are shown in figure 5.

What is clear from all these figures is that significant differences arise with respect to the trend of f_A and Λ_q / Λ_p as the basin area increases.

The explanation of the differences passes through the evaluation of the water loss attenuation with area in these regions. In order to interpret this results we address the reader to *Fiorentino and Iacobellis* (2001); they, based on the use of the Philip's equations to model water losses after surface saturation, identified three characteristic behaviors for the scaling relationship between f_A and A.

In particular, in arid regions, where significant water losses are due to initial storage and soil moisturizing, f_A was indicated to be linearly related to the basin area A raised to the power $-\varepsilon'$. In humid areas, where water losses during a flood event are mainly given by the infiltration rate at the saturation state, f_A is expected to be constant. The intermediate case is provided when a significant part of water losses are related to infiltration prior saturation. In this case,

f_A depends on the soil moisture at the time of the storm and is in turn related to long-term climatic conditions.

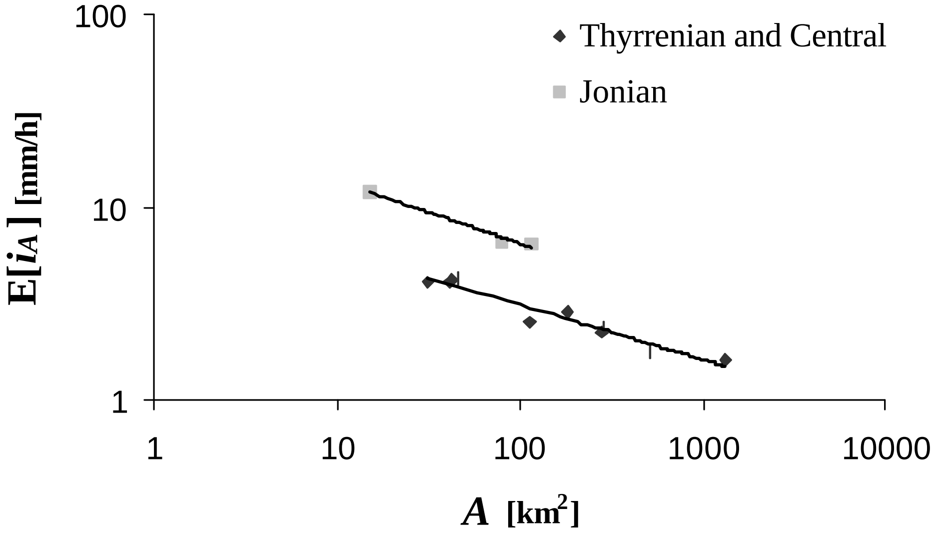

Figure 2. *Average space-time rainfall intensity vs basin area A.*

In Calabria, f_A is almost constant (figure 4). Some consistent deviations from this behavior may be observed in small basins (say below 50 km^2) where local heterogeneity may lead to consistent oscillations of the runoff threshold. In the figure, the horizontal line represents a typical behavior of humid basins where water losses do not scale with area. The constant value represented by the horizontal line is quite higher with respect to those observed in humid basins of Basilicata and Puglia (figure 5) where basins with similar climatic conditions in terms of mean annual precipitation and temperature are considered. This may reflect the fact that basins in Calabria are characterized by higher mean permeability compared to basins in Basilicata.

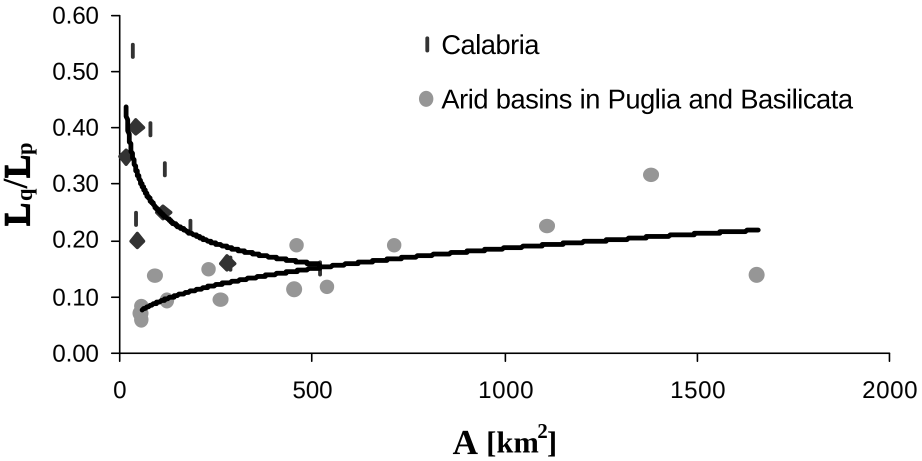

Figure 3. *Mean annual number of flood and rainfall events ratio versus basin area A in Calabria and arid basins in Puglia and Basilicata*

The observed pattern of the ratio L_q/L_p is then explained by combining equations (3) and (4), in equation (5). In facts, the slope of the scaling relationship (3) of areal rainfall intensity is substantially homogeneous over the three regions, and the resulting behavior of the ratio L_q/L_p depends on the scaling functions (4).

In light of these results, humid basins in Basilicata present similar behavior (figure 5), showing constant (yet lower than in Calabria) f_A that (compared with the regime of extreme rainfall) does not allow one to interpret consistently the results obtained in terms of the relation L_q/L_p as a function of A. On the other hand, in the case of Basilicata, the climatic control makes results intelligible (figure 6). The concept is: when associating relatively low f_A with humid basins, the result is a soil often close to saturation and a consequent high L_q/L_p. Also, when potential losses are associated to humidity, they can explicate their effect of cutting small peaks, so that a consistent reduction of flood events (with respect to rain storms) can be observed. The same behavior is not recognizable in Calabria where the flood-rainfall yield mechanism is more significantly controlled by the soil permeability than by the climate.

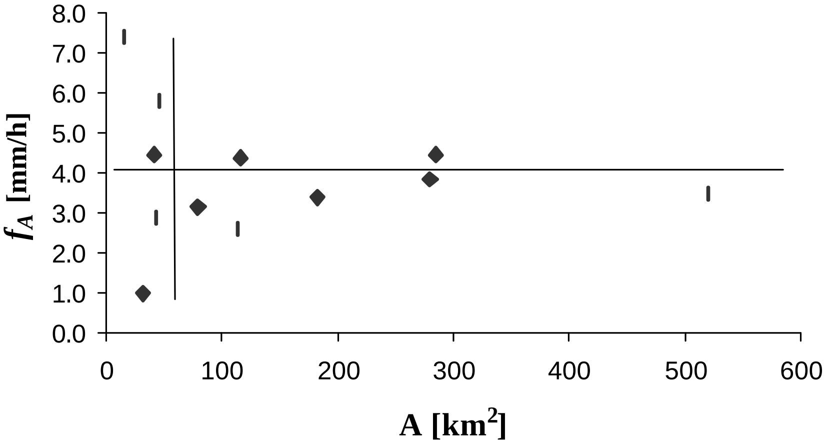

Figure 4. Average space-time water loss intensity versus basin area A in Calabria.

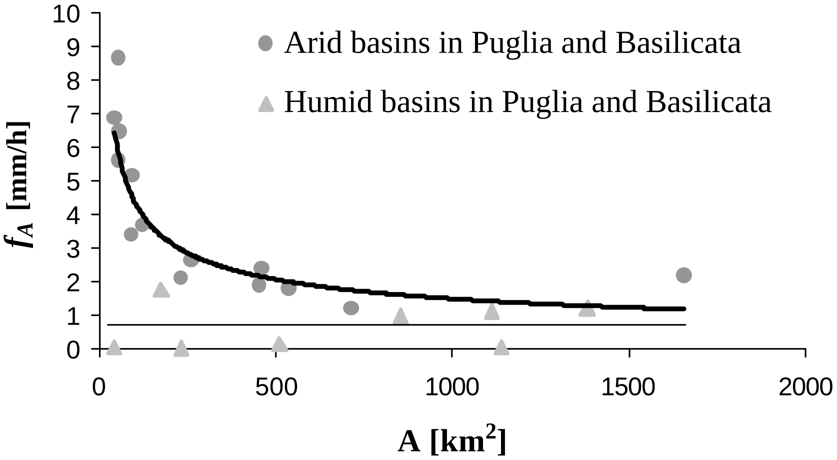

Figure 5. Average space-time water loss intensity versus basin area A in Puglia and Basilicata.

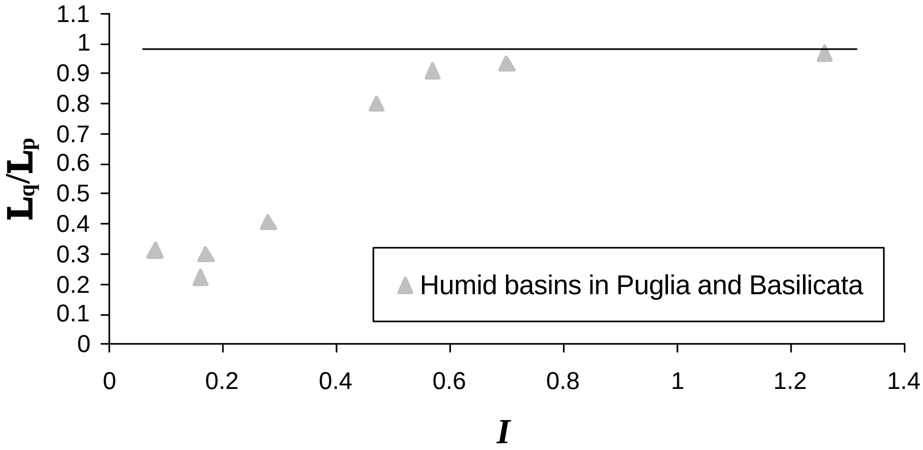

Figure 6. *Mean annual number of flood and rainfall events ratio versus climatic index I in humid basins in Puglia and Basilicata*

Acknowledgements

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