AN ANALYTICAL INDEX FOR FLOOD ATTENUATION DUE TO RESERVOIRS

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ABSTRACT

Progress on practical problems such as quantifying flood impact on piedmont regions demands increased understanding of how artificial and natural reservoirs modulates the properties of wave propagation. In particular, how dams embedded into the river network impact the peak attenuation of the flood wave remains a problem that received surprisingly little attention. In fact, although an appropriate setting and management of the dams network may offer one of the most promising tools to control flood waves propagation on mountain regions, progress on this topic is hindered by the lack of hydrology data and adequate management policy.

Here, we investigate analytically how the main features of a single dam, such as the lake surface and the width of the spillways, along with the basic characteristics of the river basin upstream and of the meteorological event, impact the flood peak attenuation. From this analysis a new synthetic index for flood attenuation, SFA*, is derived. To test the reliability and the robustness of the new index, we apply the* SFA *to a number of artificial reservoirs constituting a complex river network system in the Toce basin (North Italy).*

1 INTRODUCTION

Despite the progresses in engineering practice and management approach, flood mitigation continues to constitute a major challenge in both advanced and developing countries. In particular, current climate change predictions and increasing exploitation of floodplains suggest that extended areas in the future will experience a greater frequency of flooding.

Along with other flood management strategies, natural and artificial flood storage is now being considered as a reliable method for flood risk mitigation. Although it is surely not a panacea, flood storage could be a useful complement to conventional flood defences, particularly in the piedmont regions of the Alps. Along this line, there is now increasing interest in multipurpose use of existing reservoirs, based on the understanding that these reservoirs may provide sustainable attenuation of flood wave peak. In fact, regardless of the individual functions, they cause important changes in the downstream river regime. In particular, artificial reservoirs usually exert a partial and positive effect protecting downstream areas from flood damage.

Despite the relevant role of artificial reservoirs in flood risk mitigation, it is not surprising that a low proportion of dams are reported as having flood attenuation as their original purpose. This implies that the efficiency of existing dams in damping the flood wave peak has seldom been optimized and is often not even known. In general, in understanding and quantifying the effectiveness of a single dam in reducing flood wave peak the simple and intuitive continuity equation is the only needed mathematical tool,

$$q_i(t) - q_o(H(t)) = \frac{dV(H(t))}{dt}. \tag{1}$$

where q_i is the incoming flow discharge, q_o is the outlet flow discharge, V and H are the lake volume and the water surface level upstream the dam structure, respectively. Despite the simplicity of the hydraulic formulation, the flood attenuation problem is complicated by a plethora of variables that have to be taken into account in solving the continuity equation. These parameters are both typical of the dam structure (i.e. the link between $H(t)$ and both $V(t)$ and $qo(t)$), and intrinsic in the hydrology of the basin upstream the dam (i.e. $q_i(t)$). The overall impact of artificial reservoirs on individual floods depends on both the storage capacity of the dam (relative to the flood volume) and the way the dam is operated. There are several problems in operating a dam for flood storage purposes: a) firstly, dams are rarely used for flood storage alone and the optimum strategy for flood storage and other purposes can conflict. In general, the ideal is to keep the reservoir full and, to provide the maximum efficiency, to put it down in a drawn state in case of flooding; b) generally, water can be passed through a dam to the river downstream by at least two means: by an outlet structure at a low level and by a spillway at a higher level. The latter type of hydraulic structure may be gated to allow full control of the discharge. How to manage both the outlet structure and the spillway during an extreme event remains an indeterminate, yet crucial, problem in flood risk management.

In literature there are several interesting attempts to develop and apply complex models to a number of real cases of flood propagation (*López-Moreno et al.*, 2002; *Montaldo et al.*, 2004). All these models were developed to take into account a number of hydrological and hydraulic variable as large as possible. This approach, based on a wider and wider comprehension of the flood wave propagation, is surely valid and promising. However, the time and the length scales required for modeling wide and complex rivers-dams systems can exceed the computational ability of these type of models. In this sense, empirical and analytical models, although much simpler and more rudimental, can be more suitable to quantify the impact of a large number of reservoirs on modulating flood waves.

Some attempts to build synthetic flood attenuation indexes, both qualitative and quantitative, have been carried out in the past. In particular, it is worth citing the $FARL$ index (Flood attenuation by reservoirs and lakes) proposed by *Scarrott et al.* (1999). Being qualitative, such an index can not be considered a flood attenuation coefficient. More recently, *Piga et al.* (2000) proposed an empirical index based on a multi-regressive technique taking into account few variables, such as the area of the reservoir and of the upstream basin, and the length of the spillway gates. Because of the empirical derivation, this index is strictly applicable to basins having features comparable to those of the basins used in the calibration study.

Having in mind the necessity of improving the preliminary, large-scale, analysis on the reservoirs effects on floods, in this paper we analytically investigate how the main features of a single dam, (e.g. its total area, the width of the spillways, the basic river basin characteristics and the intensity of meteorological events), impact the flood peak attenuation. To accomplish this goal, a number of assumptions regarding the way the dam is operated have been adopted: a) we use a conservative and prudential approach that considers the reservoir always completely full at the spillway crest level; b) for safety reasons and analytical manageability, we assume the outlet structures closed and all the spillways ungated, so that no gate management must be handled. We are aware that these are very restrictive assumptions; nevertheless we are interested in building a rather general and robust index, that can describe some objective flood attenuation potentiality of existing reservoirs under real (i.e. non-optimal) conditions. As stated before, the management optimization of existing dams is a much broader topic and is beyond the scope of this paper.

2 THE STATE OF ART

In literature, synthetic indexes for flood attenuation are build upon simple hypothesis about flood waves shape and spillway typology. These indexes can be classified in qualitative and quantitative. Qualitative synthetic indexes provide information about flood attenuation effects for basins without difference between artificial or natural lakes, to allowing comparison between different reservoirs. Quantitative synthetic indexes provide coefficients based on few significant parameter as reservoir storage volume, flood volume, flood peak time and duration. A qualitative synthetic index is the Flood Attenuation by Reservoirs and Lakes (FARL) (*Scarrott et al.*, 1999), which is a relation between reservoir water surface A_L, basin surface A_B and catchment area A_C. In this index the attenuation is $\alpha = (1 - \sqrt{r})^w$ where $r = A_L/A_B$ and $w = A_B/A_C$. In basins containing more reservoirs, the FARL index is the product of different α estimated for every lake. This index has been built as it was a geomorphoclimatic parameter having an influence on the index flow, which in the FEH ((*AA.VV.*, 1999)) is calculated as

$$Q_{index} = a \prod X_i^b, \tag{2}$$

where X_i are geomorphoclimatic parameters and a, b are coefficients to be estimated by regression.

For the design of a reservoir storage capacity, so with flood attenuation know a priori, a quantitative index proposed by *Mays* (????). This index is based on the shape of the inflow hydrograph and on a given maximum water elevation in reservoir. A promising way to evaluate flood attenuation indexes is based on multiregressive approaches. For example, *Piga et al.* (2000) calculated a flood attenuation index using multiregressive analysis based on parameters as $L[m]$ (spillway width), $A_L[km^2]$ (lake area) and $A_B[km^2]$ (basin area). The authors obtained the following form of a flood attenuation index

$$\eta = 1 + \exp[0.119\ L^{0.225}\ A_L^{-0.583}\ A_B^{0.405}] \tag{3}$$

valid for high return periods and for basins with climatical and hydrological characteristics similar to the Sardinia basins, used to estimate the coefficients in equation 3.

3 THEORY BEHIND A NEW INDEX

The overall efficiency of a reservoir in attenuating flood peaks is usually expressed through the ratio $\eta = Q_o/Q_i$, where $Q_o = Max(q_o(t))$ and $Q_i = Max(q_i(t))$. Hence, the core of the efficiency problem is to evaluate the behavior of q_o and q_i. This can be done, as stated before, through the continuity equation once q_i (i.e. the incoming hydrograph) and the mathematical relations $q_o(H)$ and $V(H)$ are known. While the evaluation of q_i is an hydrological problem, $q_o(H)$ and $V(H)$ are of easier evaluation. In particular, $V(H)$ simply depends on the storage volume geometry and is usually parameterized through a power law:

$$V(H) = V_o + A_L\ H^m \tag{4}$$

where V_o and A_L are the storage volume and the lake area when the water level is at the spillway crest, respectively, H is the water depth over the spillway crest, and m is a fitting coefficient. The value $q_o(H)$ depends both on the typology of outlet structures and on the way they are operated. As stated before, we assume: a) the initial water level coinciding with the lowest crest of the available spillways, (i.e. the flood storage volume is only the surcharge storage), b) outlet structures at a low level are considered closed; only the spillway gates are used to evacuate the incoming flood, and c) the reservoir is not "actively" governed: it passively impact the flood wave through ungated spillways. Under these conditions $q_o(H)$ can be written as

$$q_o(H) = c_f \; L \; H^r \tag{5}$$

where c_f is a coefficient of discharge, L is the total length of the spillways, and $r = 3/2$. From equations (4) and (5) we get

$$V(t) = V_o + A_L \left[\frac{q_o(t)}{c_f \; L} \right]^{\frac{m}{r}} \tag{6}$$

Here we introduce the additional condition of linear reservoir, that assumes the equality between the two coefficients m and r. To this end it is to consider that m varies, at least for alpine reservoirs, between 1.2 and 1.6, then making the linearity a plausible assumption. Based on these parametrization, the continuity equation becomes

$$q_i(t) - q_o(t) = I \frac{dq_o(t)}{dt}. \tag{7}$$

that is an ODE having a general solution of the type:

$$q_o(t) = e^{-\frac{t}{I}} \left(C + \frac{\int^t e^{\frac{t_d}{I}} q_i(t_d)}{I} dt_d \right), \tag{8}$$

where $I = A_L/(c_f \; L)$, t_d is a dummy variable, and C is the integration constant. Note that this solution does not constrain the incoming flow to have a prescribed form. Nevertheless, to obtain a simple solution of equation (8), as the base to build a viable attenuation index, we prefer to simplify $q_i(t)$ as much as possible while retaining the fundamental information of a real flow hydrograph (i.e. the volume and the duration of the flood). In order to clearly describe the procedure used to evaluate $q_i(t)$, a brief review on synthetic hydrographs is provided below.

4 PARAMETERIZATION OF THE INCOMING HYDROGRAPH

Plausible and robust assumptions on the incoming hydrograph represents perhaps the most sensitive part of a reliable flood attenuation analysis. In fact, even though the volume and the peak are given, the shape variability of $q_i(t)$ may markedly impact the evaluation of reservoirs efficiency in attenuating a flood. A possible general hydrograph shape, hereafter called PR hydrograph, can be written as $q_i(t) = Q_c \; (\epsilon_D \; + t \; \partial_D \; \epsilon_D)$, where Q_c and ϵ_D are the value and the reduction ratio of the flood peak discharge. The reduction ratio, as a function of a generic hydrograph duration D, is defined as

$$\epsilon_D = Q_D/Q_c \qquad \text{where} \qquad Q_D = \max\left(\frac{1}{D} \int_t^{t+D} q_i(\tau) d\tau \right). \tag{9}$$

and it should characterise synthetic flood waves analogous to observed ones.

This approach, exemplified in figure (1) is very useful, being ϵ_D parameterizable both analytically (*Bacchi et al.*, 1992; *Fiorentino et al.*, 1987) and empirically *AA.VV.* (1975) once the hydrological features of the basin are known. For example, *Pianese* & *Rossi* (1986) suggested

$$\epsilon_D = (1 + b\, D)^{-c}, \tag{10}$$

where $b = 1/(2\, t_R)$, t_R is the basin delay time, $c = 1 - n$. Based on this definition for ϵ_D the expression of *PR* becomes

$$q_{cen}(t) = Q_c[(1 + 2\, b\, |t - t_p|)^{-c} - 2\, b\, c\, |t - t_p|\, (1 + 2\, b\, |t - t_p|)^{-c-1}], \tag{11}$$

where Q_c and t_p are value and position of the flood peak . Note that the hydrograph parameterized using equation (10) is symmetric with respect to the peak.

A simplified synthetic hydrograph: The PR hydrograph can be simplified neglecting the time dependence as $q_i(t, D) = Q_c\, \epsilon_D$, where the incoming flow is now also variable with D. This simplification, produce rectangular form that, while making q_i easier to be treated mathematically, allow to carry out a maximizing approach on the efficiency by varying the duration D. This is done in the next section.

5 THE SYNTHETIC FLOOD ATTENUATION INDEX

Having defined the simplified hydrograph, the outgoing hydrograph (i.e. equation (8)) can now be written, as a function of t and D, as

$$q_o(t, D) = Q_c\, e^{-\frac{D}{K}} \left(1 - e^{-\frac{t}{I}}\right), \tag{12}$$

where K and $I = A_L/(c_f\, L)$ take into account the main hydrological and hydraulic variables. In this expression, instead of equation (10), a simpler parametrization of the reduction ratio, $\epsilon_{DS} = \exp(-D/K)$, has been used. In this formulation, the parameter $K = 1/(b\, c)$ can be derived equating the linear series expansion of equation (10) and ϵ_{DS}.

To evaluate the efficiency of the reservoir in attenuating the flood peak, the maximum flow discharge and the relative flood duration have to be defined from equation (12). In figure 5 both $q_i(t, D)$ and $q_o(t, D)$ are shown for several duration, D. Note that, for each $q_i(t, D)$ the associated $q_o(t, D)$ is a monotonic growing function and its maximum is always at $t = D$. Figure 5 also shows the function obtained merging the maximum value of $q_o(t, D)$ (i.e. the so-called $q_o(D, D)$). Note that for small D the inflow is very large but the flood volume is limited. On the other hand, for large D the peak is modest and the volume is very large. In the first case, being the flood volume modest, the reservoir efficiency is very high but, because of the large peak, the maximum is still considerable. On the contrary, in the second case the efficiency is scarce but, being the peak already very small, the maximum outflow is also very small. This behavior implies that the maximum outlet flow is for an intermediate duration, D_C, for which both the flood volume and the wave peak are still relevant. From a quantitative perspective, we analytically get the absolute maximum of $q_o(D, D) = max[q_o(t, D)]$ as

$$q_o(D_c) = max[q_o(D, D)] = Q_c\, e^{-\frac{D_c}{K}} \left(1 - e^{-\frac{D_c}{I}}\right) = Q_c\, \frac{K}{I + K} \left(\frac{I + K}{I}\right)^{\frac{K}{I}}, \tag{13}$$

where $D_c = I \log[(I + k)/I]$ is the critical flood duration for which the flood attenuation is minimized. In comparison to the solutions obtained by *Pianese* & *Rossi* (1986) and by *Fiorentino* & *Margiotta* (1997) relation 13 allows one to obtain a next simplified attenuation index (SFA) by

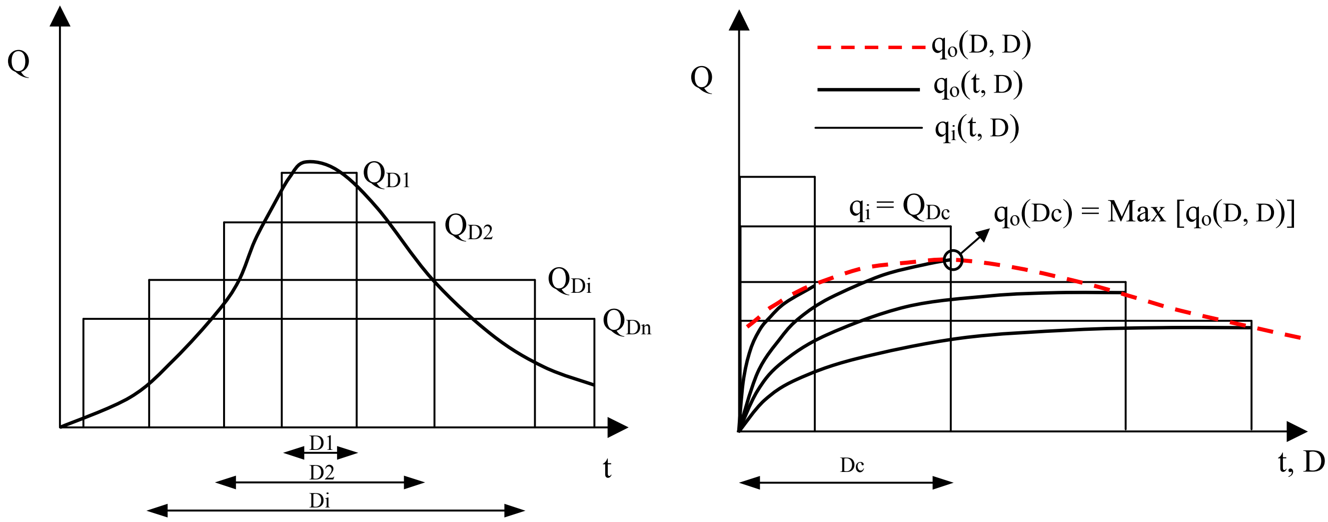

Figure 1: Left panel: Rectangular synthetic hydrographs as defined in equation 9. Right panel: Representation of incoming hydrographs, $q_i(t, D)$, along with the associated outlet flow, $q_o(t, D)$. The maximum outflow and the relative critical duration, D_C, are also shown.

$$SFA = \frac{1}{R+1}, \qquad \text{where} \qquad R = I/K = \frac{A_L\,(1-n)}{2\,t_R\,c_f\,L}. \tag{14}$$

Analyzing qualitatively the SFA index one can note that the larger the storage area (i.e. A_L) the smaller the SFA. Similarly, having assumed the reservoir unlimited, the narrower the spillway gate the smaller the SFA. This is coherent with what expected; in fact, small spillways and wide storage volumes have the effect of retaining upstream larger amount of water, then reducing the flood peak. On the contrary, large coefficient n and wide basin area involve great flood volumes, hence reducing the efficiency of a given reservoir. In the next section a more quantitative analysis is carried out.

6 SFA VALIDATION VIA A CASE STUDY

To check the reliability of the SFA index, a sensitivity analysis has been carried out using a number of different synthetic hydrographs and sixteen artificial reservoirs existing in the Toce basin, as described below.

The Toce basin is an alpine basin located in the North of the Piemonte region, in Italy. It has also the 10% of its area in the southern Switzerland. The total area is about 1780 km^2, and the main stream length is of 80 km, estimated from its sources near San Giacomo Pass, to the Maggiore Lake. This basin is characterized by steep narrow valleys with glacial origin. Elevations within the basin range from 4623.5 m a.s.l. (Rosa Mountain) to 193 m a.s.l. of the Maggiore Lake, with an average of about 1500 m s.l.m.. Climate within the basin is influenced by the high elevations and by heavy precipitations. The Toce basin contains sixteen dams, whose main features are shown in Table (1). Given that these reservoirs are not a significant group to constitute a robust case study, we have considered a much higher number of virtual reservoirs, changing at random the six features shown in Table 1, and removing only the least reasonable resulting configurations. These virtual reservoirs, along with the real ones, have been used to check the capability of the SFA index to evaluate the reservoir efficiency in attenuating floods.

Table 1: Principal characteristic of the sixteen artificial reservoirs in the Toce basin.

RESERVOIR	$A_B[km^2]$	$t_R[ore]$	$n[-]$	$Q_m[m^3/s]$	$A_L[km^2]$	$L[m]$
Agaro	10.60	0.556	0.51	19.55	0.650	36.00
Agrasina	17.83	1.128	0.50	31.66	0.0135	12.30
Alpe Cavalli	23.74	1.253	0.56	29.03	0.467	18.00
Alpe Larecchio	3.02	0.464	0.50	6.90	0.183	25.00
Busin Inferiore	2.54	0.357	0.46	6.13	0.336	10.00
Camplicciolí	35.19	1.401	0.56	38.40	0.324	13.50
Camposecco	4.08	0.497	0.56	7.28	0.383	105.85
Ceppo Morelli	121.00	2.099	0.56	97.248	0.040	84.00
Devero	25.38	0.866	0.51	36.40	0.96	25.50
Lago Avino	5.32	0.497	0.54	9.04	0.43	45.25
Lago Cingino	3.06	0.431	0.56	5.78	0.14	65.00
Morasco	35.26	1.305	0.46	42.59	0.65	45.00
Quarazza	25.81	1.002	0.55	35.26	0.028	31.20
Sabbione	14.37	0.818	0.46	22.93	1.21	10.00
Val Toggia	10.32	0.720	0.44	17.71	0.81	28.00
Vannino	11.94	0.727	0.47	20.75	0.48	22.80

To carry out the sensitivity analysis, a number of different synthetic inflow hydrographs are considered, to which numerically implement the continuity equation. Here we use both triangular and PR hydrographs. For each of these forms we can use the canonical expression (i.e. with a central peak) or similar hydrographs having the peak at $t = 0$ and $t = D_{max}$, where $D_{max} = 2\, t_p$ is the flood duration. The PR hydrograph can be respectively written as

$$q_{in}(t) = Q_c[(1 + b\,t)^{-c} - c\,b\,t\,(1 + b\,t)^{-c-1}], \tag{15}$$

$$q_{fin}(t) = Q_c[(1 + b\,|t - t_{max}|)^{-c} - b\,c\,|t - D_{max}|\,(1 + b\,|t - D_{max}|)^{-c-1}]. \tag{16}$$

In Figure (6) the three PR forms and the three triangular ones are shown. Note that both the volume and the value of the peak discharge are constant for the six hydrographs. Using the continuity equation (i.e. equation 1) six values of η_i have been calculated for each reservoir: η_1, η_2, η_3 are for the triangular hydrographs having, respectively, right, central, and left peak. Similarly, η_4, η_5, η_6 are the efficiencies for the PR hydrographs.

The efficiency value obtained, η_i, along with the SFA index, shown in Figure (6) as a function of the mean efficiency (obtained from the six η_i). From, Figure (6) it is apparent that the maximum and the minimum value of η are very different for each reservoir. This finding demonstrates the sensitivity of the reservoir efficiency to the hydrograph shape. Moreover, for each synthetic hydrograph, the flood wave attenuation presents a well defined behavior. Not surprisingly, while the worst efficiency is always for the triangular hydrograph having the peak at $t = t_p$ (η_1 in figure (6)) the best efficiency is for the **RP** hydrograph with the initial peak (η_6 in figure (6)). This strong sensitivity of η on the flood shape confirms the difficulty in defining an unique level of efficiency of reservoirs in attenuating floods. Nevertheless, it is encouraging that the SFA index, both for the real and the virtual reservoirs, is well included into the variability due to the hydrograph shape, without any bias. Furthermore, note that most of the SFA values are bounded by η_2 and η_5 that represent the triangular and the PR hydrograph

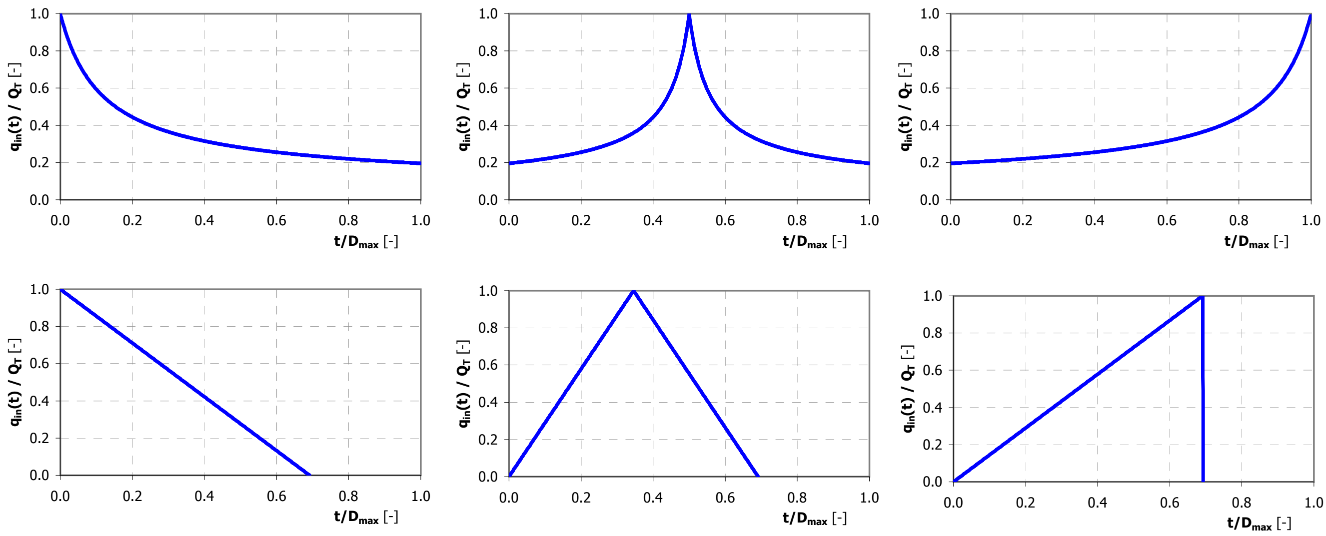

Figure 2: The six hydrographs used to carry out the sensitivity analysis.

for the central peak case. This finding is particularly encouraging being the central peak cases surely the closer description of real hydrographs. Moreover, the triangular hydrograph with a central peak is often used by the professional engineers to design or verify the volume of artificial reservoirs.

6.1 An expeditious SFA index

When one is interested in roughly evaluating the efficiency of a number of artificial reservoirs in attenuating the flood peak without knowing the specific features of both the basin and the reservoir, such as n, c_f and t_R, an expeditious index can be apply instead of the complete SFA index. Here an expeditious SFA index, hereafter called SFA_e, is defined based on the following considerations. a) The discharge coefficient can to assume a constant value; for example, we assume $c_f = 0.31$, given that most of the existing spillways have a standard Creager weir. b) For every reservoir, a standard coefficients n can be assumed (i.e. in this paper the mean value computed over the whole Toce basin has been used). c) The lag time can be taken as $t_R = A_B^{0.5}$. Using these simplifications the parameter R in equation (14) becomes

$$R = \frac{s\, A_L}{\sqrt{A_B}\, L}. \tag{17}$$

where s is a coefficient that takes into account the previous simplifications for n and c_f (i.e. $s \simeq 100$ in our case study). In figure 6 b the SFA_e is shown along with the actual reservoir efficiency for the six synthetic hydrographs. Note that the evaluation of η is comparable with that of the more complex SFA index. In particular, the SFA_e index is still bounded by η_2 and η_5 and does not show any particular bias.

CONCLUSIONS

Flood control through dams storage volumes represents a challenge for the application of statistical procedures related to flood risk planning. In fact, the high specific knowledge of the deterministic (hydraulic) part of the phenomenon contrasts with the lack of methods able to grossly but systematically assess performances and attitudes

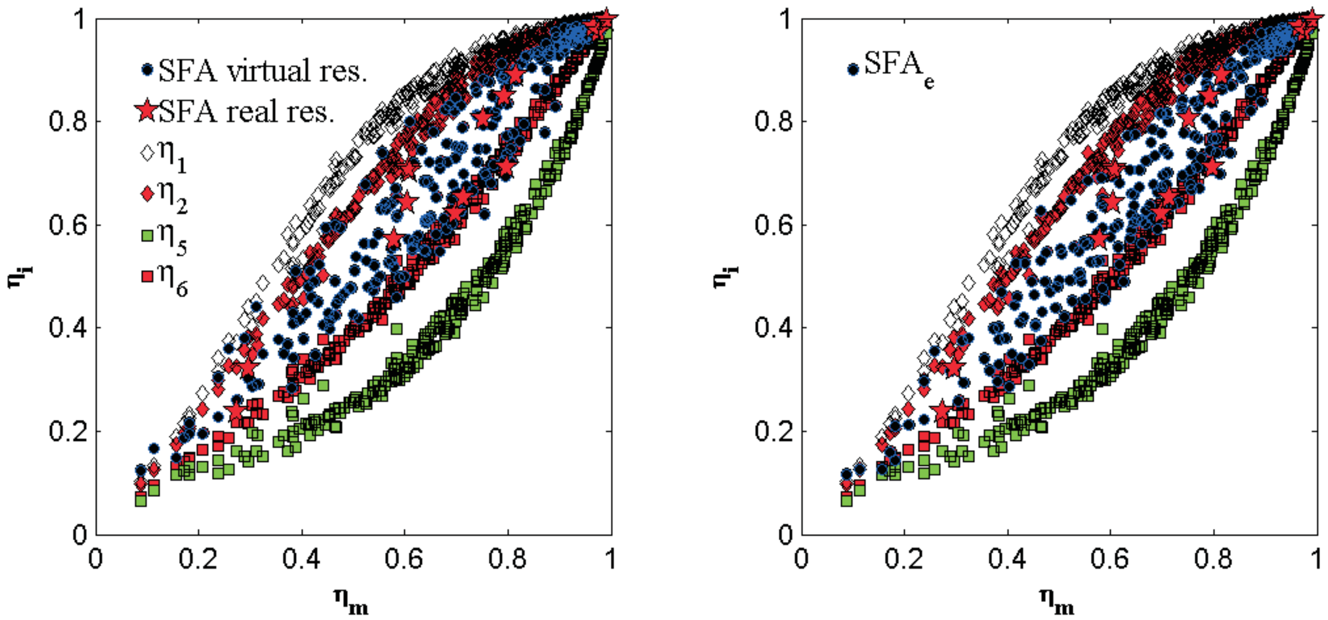

Figure 3: The real and virtual reservoir efficiency; η_1, η_2, are for the triangular hydrograph with right and central peak; η_5 and η_6 are for the PR hydrograph with central and left peak.

of systems of dams towards flood risk reduction. As a contribution toward this goal, a new synthetic index for flood attenuation, *SFA*, has been proposed here. Such an index takes into account both the main features of artificial reservoirs, such as the lake surface and the spillways width, and the basic characteristics of river basins and rainfall forcing. Despite the fact that several assumptions were used in the analytical development of the index, the directness of the approach, makes the *SFA* index an apparently robust tool to preliminarily sort out complex dam networks.

The index performances were in fact tested by means of a sensitivity analysis, in which we apply the *SFA* to a considerable number of real and hypothetical dams, built around the complex system of reservoirs in the Toce basin (Northern Italy). Each one of this reservoirs has been subjected to a number of synthetic hydrographs, characterized by different shapes but identical volume and peak, computing the reservoir efficiency in reducing the hydrograph peak, η. These actual values of η were compared with the SFA index, which is intended as a flood shape-independent value. The comparison showed that *SFA* values fall in the middle part of the η distribution range, and present no apparent bias. An equally good description of the overall flood attenuation capability of the considered dams was also demonstrated by an expeditious index, SFA_e, merely based on basin and lake area and on spillway length.

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