



A data-based assessment of the dependence of short-duration precipitation on elevation

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ARTICLE INFO

Article history:

Received 15 July 2008

Received in revised form 24 December 2008

Accepted 8 January 2009

Available online 18 January 2009

Keywords:

Orographic effect

Intense rainfall

DDF curves

Elevation

ABSTRACT

Variability of precipitation with elevation is often related to the presence of an orographic effect interacting with prevailing arrival directions of wet air masses. This effect is commonly recognized to be responsible for the increase with elevation of the annual precipitation amounts measured at the ground level. However, the variability with elevation of heavy rainfall of short duration is poorly investigated in hydrology, despite the importance of short-duration events in hydrological applications. Analyzing a database of 567 time series of annual maximum sub-daily rainfall in northern Italy, we find the relation of extreme precipitation with elevation to be a function of the event duration. In particular, it emerges that the intensity of rainfall decreases with elevation for very short durations (i.e., 1–3 h), while the negative slope of the intensity–elevation regression lines tends to decrease when considering events of longer duration (i.e., 12–24 h), at least in the western part of the alpine chain. A combined use of kriging and regression techniques is then proposed to account for the effect of elevation and longitude in the spatial interpolation of sub-daily rainfalls.

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1. Introduction

In the middle latitudes, precipitation in the winter season predominantly derives from advective air movements, (i.e., the uplift from the horizontal plane of a warm air mass). Orographic uplift is a special case of advection, where air masses are forced to rise by the relief of the land they pass over, with consequent increase with elevation of the precipitation amounts measured at the ground level (Elliott and Hovind, 1964; Foufoula-Georgiou and Georgakakos, 1991; Barry, 1992; Hevesi et al., 1992; Barros and Lettenmaier, 1994; Rotunno and Ferretti, 2001; Borga et al., 2005; Roe, 2005). Capturing the existence of these mechanisms in mountainous environment is important to understand the behavior of precipitations at high-elevation. In high-elevation regions, in fact, rainfall patterns are usually not very well known, partly because of the complex topography, and partly because of the spatial unevenness of the information available to study such relationships (Molnar and Burlando, 2008).

Several papers investigate the relation between mean annual precipitation and elevation (e.g., Basist et al., 1994; Harris et al., 1996; Guan et al., 2005), typically finding significant increasing relations between the two. A common belief that precipitation amounts increase with elevation has then formed, so that the

mountainous environment is commonly believed to be prone to more intense and frequent precipitation events.

When passing from long-duration to short-duration events the relation between precipitation and elevation is not univocal, and few literature studies exist that investigate the problem (e.g., Prudhomme and Reed, 1999; Weisse and Bois, 2001; Boni et al., 2006). In this paper we investigate this relation and find that, contrary to expectations, maximum annual precipitations of short duration on a vast alpine region situated in northern Italy significantly decrease with elevation.

A first implication of this finding is that the parameters of the depth–duration–frequency curve (DDF), which is commonly used to estimate design rainfall at ungauged sites (see e.g., Gilman, 1966; Burlando and Rosso, 1996), vary with elevation. This complicates the spatial interpolation of the coefficients of the curve. Methods for incorporating elevation into the spatial interpolation of rainfall are available, as for example the ‘kriging with an external drift’ method or the ‘detrended kriging’ method (e.g., Gotway and Hartford, 1996; Prudhomme and Reed, 1999). However, these methods have not been specifically designed to deal with extreme rainfall. In this paper we address the problem of defining a procedure to account for the effects of elevation into the spatial interpolation of short-duration precipitation, based on the combined use of kriging and regression techniques.

The study region and the data that are used in this study are described in Section 2. In Sections 3 and 4 the relationships between precipitation amounts, elevation and longitude are presented and

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the parameters are estimated. An application aimed at spatially interpolating the DDF parameters (Section 5) closes the paper.

2. The study region

The study area covers the entire alpine region of Italy, located in the northern part of the country at latitudes that vary from 44° to 47° North (see Fig. 1A). The mountain chain is about 700 km long, that is equivalent to 7.5 longitude degrees. 567 rainfall stations are considered in this study (Fig. 1B), derived from the national hydrological information system set up under the CUBIST project (see www.cubist.polito.it). Each station j is identified by its longitude, latitude and elevation (x_j, y_j, z_j) and by the historical record of maximum annual precipitations $(h_1, h_3, h_6, h_{12}, h_{24})$ at different durations (1, 3, 6, 12 and 24 h), over a time span going from 1930 to 1990. In this study only the alpine and pre-alpine stations having series of at least 10 years of data and with an elevation greater or equal to 200 m a.s.l are considered. In Fig. 1C the distribution of the stations with elevation is shown.

3. Model formulation

The relation among depth, duration and frequency of precipitation of short duration is represented with different mathematical expressions in different regions of the world (eg., Bell, 1969; Koutsoyiannis et al., 1998). In Italy, a power law expression is commonly used to relate the mean of the maximum annual intense precipitation, h_d , to the rainfall duration d (see e.g., Burlando and Rosso, 1996)

$$h_d = ad^n, \quad (1)$$

where the coefficients a and n are estimated from the data collected at the station. A relation between the depth–duration curve and the return period T is then obtained according to a frequency curve, by introducing in Eq. (1) a multiplicative factor (growth factor) expressed as a function of T (e.g., scaling model of Burlando and Rosso (1996)). In this paper the dependence of the parameters a and n on elevation is analyzed, in order to investigate the variability of the

average DDF curve (i.e., corresponding to the mean annual extreme precipitation of duration d) with elevation. In contrast, we will not consider the variations with the return period of the depth–duration curve.

A necessary condition for the construction of the DDF at a point is that historical series of maximum rainfall intensities h_d for different durations are available at the station. This allows one to evaluate for each station the parameters of the DDF by linear regression of $\ln h_d$ versus $\ln d$. In the following subsection the variability of the coefficients a and n with elevation (Section 3.1), longitude (Section 3.2) and latitude (Section 3.3) are separately considered.

3.1. Elevation

In this section a statistical model to quantify the variations, if any, of the DDF curve coefficients with elevation is proposed. We relate a to the elevation z by means of a power law model

$$a = a_0 z^{a_1}, \quad (2)$$

while a logarithmic model is used for $n(z)$:

$$n = n_0 + n_1 \ln z. \quad (3)$$

These specific mathematical formulations of the $a(z)$ and $n(z)$ relations are adopted to have a model that congruently represents the $h_d(z)$ relation as a power law. In fact, substituting Eqs. (2) and (3) into (1), with simple manipulations one obtains

$$h_d = a_0 d^{n_0} \cdot z^{a_1 + n_1 \ln d}, \quad (4)$$

that, for any fixed duration (e.g., $d = 1$ h), has the same form of Eq. (2).

3.2. Longitude

As mentioned, the study region is 7.5 longitude degrees wide. For this reason one can suppose that a_0 , a_1 , n_0 and n_1 in Eqs. (2) and (3) are also functions of the longitude x . The models for $a_0(x)$, $a_1(x)$, $n_0(x)$ and $n_1(x)$ are designed to maintain the linearity

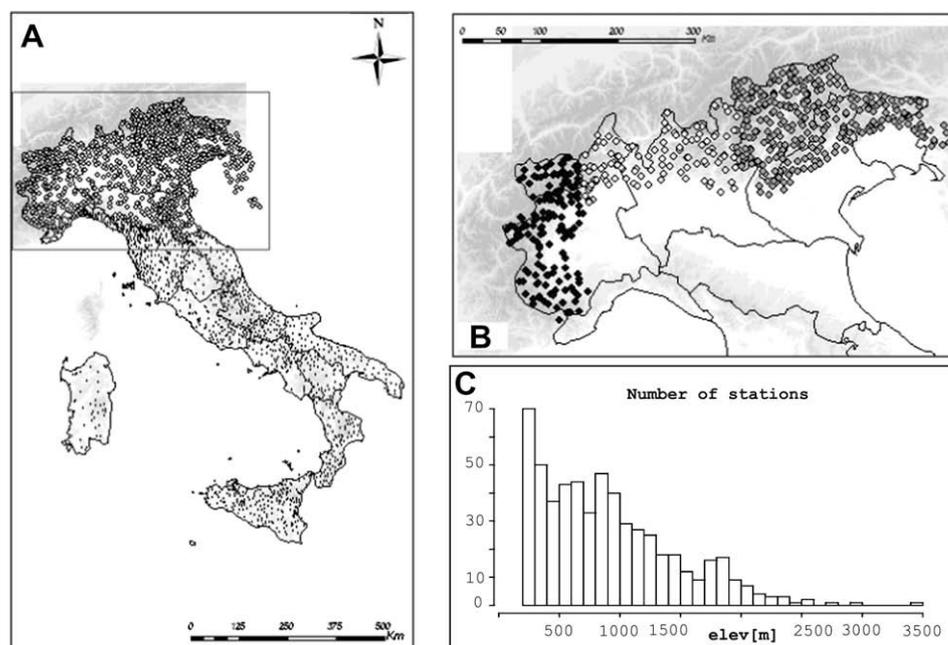


Fig. 1. Panel A: map of the rainfall stations available in the CUBIST database. Panel B: zoom on the alpine area of Italy, where different gray shades refer to the three geographic areas considered (western, central and eastern Alps). Panel C: frequencies of the station availability in the Alps at the different elevations.

of the relations $\ln a(z)$ and $n(z)$. For what concerns n_0 and n_1 , it is easy to verify that the simplest conjectures

$$n_0 = n_{01} + n_{02}x, \tag{5}$$

$$n_1 = n_{11} + n_{12}x, \tag{6}$$

guarantee that the relation $n(z,x)$,

$$n = n_{01} + n_{02}x + n_{11} \ln z + n_{12}x \ln z, \tag{7}$$

is linear in the coefficients, which allows one to use the least squares method for estimation (while, for example, a model $n_1 = n_{11}x^{n_{12}}$ would not have this property). Another property of this model is that it is unaffected by the re-scaling of the explicative variables. This means, for example, that one can divide x by an arbitrary constant \bar{x} (or z by \bar{z}) without modifying the structure of the model.

On the contrary, using linear models for $a_0(x)$ and $a_1(x)$ (analogous to the ones in Eqs. (5) and (6)) would imply a complex non linear dependence $a(z,x)$. To overcome this problem $a_0(x)$ and $a_1(x)$ are expressed as

$$a_0 = a_{01}x^{a_{02}}, \tag{8}$$

$$a_1 = a_{11} + a_{12} \ln x. \tag{9}$$

Substituting the above relations in Eq. (2) and taking the logarithms one obtains

$$\ln a = \ln a_{01} + a_{02} \ln x + a_{11} \ln z + a_{12} \ln x \ln z, \tag{10}$$

which is again a linear model in the coefficients. Observe that in Eq. (9) we use as explicative variable $\ln x$ instead of x in order to preserve robustness of the model in (10) with respect to normalization. If not, the model would be sensitive to a normalization x/\bar{x} .

3.3. Latitude

The model we propose, in contrast, does not account for the variations, if any, of a and n with latitude for two reasons. The first is that the latitude range covered by the region of study is smaller than the longitude range. The other reason is due to the geomorphology of the Italian Alps, where latitude is significantly correlated to elevation, with higher elevations corresponding to the

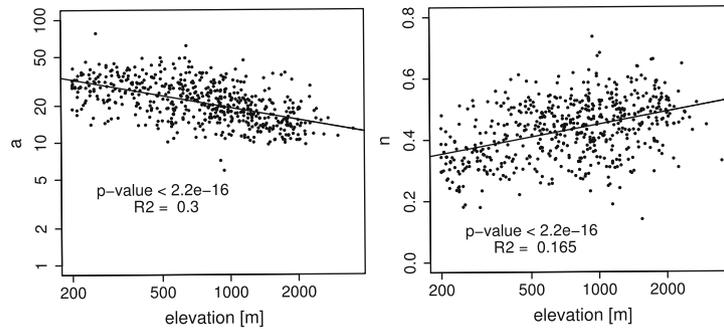


Fig. 2. Dependence of the DDF curve parameters, a and n , on elevation for the 567 considered stations.

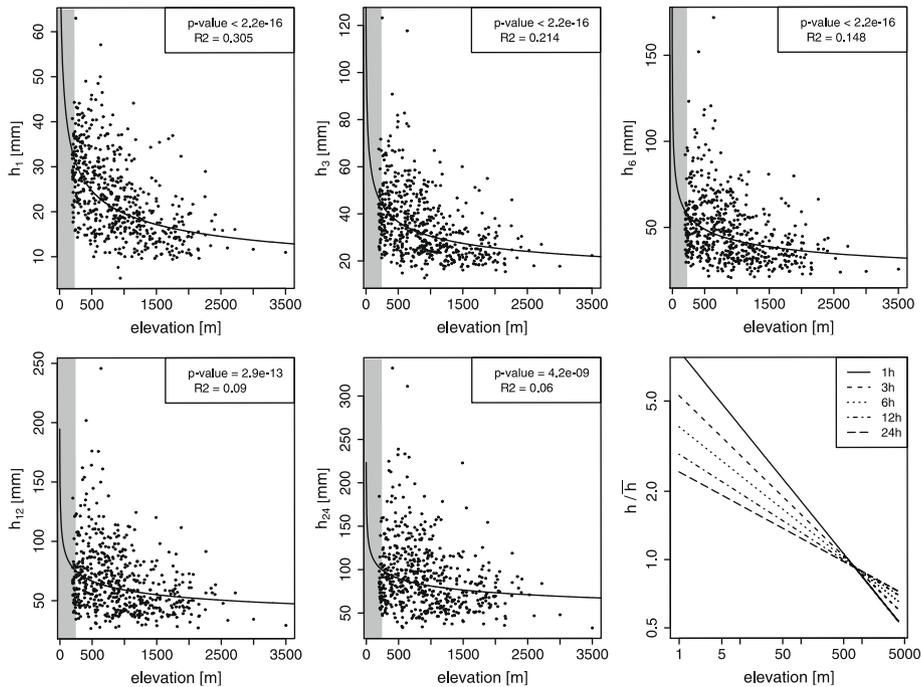


Fig. 3. Dependence of average maximum rainfall of duration d , h_d , on elevation for the five considered durations. The graph in the bottom right corner shows the regression curves on a bi-logarithmic plane. No data are considered in the grey shaded area.

northernmost points. We have verified that using two mutually correlated explicative variables would negatively affect the robustness of the model.

4. Model application

The values of the parameters of the DDF curves are estimated for each station from the historical records of maximum rainfall intensities, obtaining suitable estimators for a_j and n_j (with $j = 1-567$). The coefficients in Eqs. (2), (3), (7) and (10) are then estimated by linear regression. For both the univariate formulations (2) and (3), a significant dependence of a and n on z is found. In particular, it is observed that the coefficient a decreases with elevation while n increases (see Fig. 2A and B, respectively). The curves corresponding to the $h_d(z)$ model in Eq. (4) are represented for the five durations in Fig. 3. It emerges that the rainfall depth significantly decreases with elevation for very short durations (i.e. 1–3 h), while the negative slope of the depth–elevation regression curves decreases when considering events of longer duration.

For the bivariate model $a(z,x)$, the estimated values of the regression coefficients are listed hereafter, with the correspondent p -values in parenthesis

$$\begin{cases} \ln a_{01} = 5.25 & (p \cong 0), \\ a_{02} = 0.57 & (p = 0.26), \\ a_{11} = -0.33 & (p \cong 0), \\ a_{12} = -0.06 & (p = 0.43). \end{cases} \quad (11)$$

The coefficients are referred to a model in which longitude is expressed in UTM coordinates divided by the sample mean longitude $\bar{x} = 578,430$, in order to have more manageable values. It emerges that in Eq. (10) the terms a_{02} and a_{12} are not statistically significant (p -values of 0.26 and 0.43, respectively), while a_{11} results to be highly significant. This means that the coefficients a vary only with elevation, the slope of this curve remaining nearly constant throughout the region. This is exemplified by the graphs in the first row of Fig. 4. In these graphs the sample has been further divided into three sub-samples, configuring an ideal distinction between western, central and eastern Alps (with different grey shade in

Fig. 1B). For these three sub-regions, a similar behavior of the coefficient a with elevation is observed (i.e., in the bi-logarithmic plane the slopes of the lines remain nearly the same); this implies that the $a(z)$ relation does not vary significantly from the West to the East, as confirmed by the non significance of the terms in x in Eq. (10). On these bases the conjectures formulated in Eqs. (8) and (9) have to be rejected and the coefficients a_0 and a_1 have to be kept constant according to the univariate model in Eq. (2). Therefore, by linear regression one obtains

$$\begin{cases} \ln a_0 = 5.21 & (p \cong 0), \\ a_1 = -0.33 & (p \cong 0). \end{cases} \quad (12)$$

Analogously for n (Eq. (7)) one obtains

$$\begin{cases} n_{01} = -0.84 & (p = 7.7 \times 10^{-9}), \\ n_{02} = 0.91 & (p = 4.6 \times 10^{-10}), \\ n_{11} = 0.19 & (p \cong 0), \\ n_{12} = -0.14 & (p = 2.2 \times 10^{-10}). \end{cases} \quad (13)$$

In this case all the regression coefficients are statistically significant, meaning that the dependence of n on elevation also varies with longitude. This is exemplified by the graphs in the second row of Fig. 4, that show how the slope of the regression lines changes from western to eastern Alps.

One can combine the previous results to derive the complete model for $h_d(x,z)$ where, since the dependence of a on x was found to be not statistically significant, the coefficients a_0 and a_1 are kept constant (see Eq. (12)). With simple manipulations, one obtains a bivariate expression that relates h_d to x and z

$$\ln h_d = \ln a'_0 + a'_1 \ln z + n'_0 x + n'_1 x \ln z, \quad (14)$$

where

$$\begin{cases} \ln a'_0 = \ln a_0 + n_{01} \ln d, \\ a'_1 = a_1 + n_{11} \ln d, \\ n'_0 = n_{02} \ln d, \\ n'_1 = n_{12} \ln d, \end{cases} \quad (15)$$

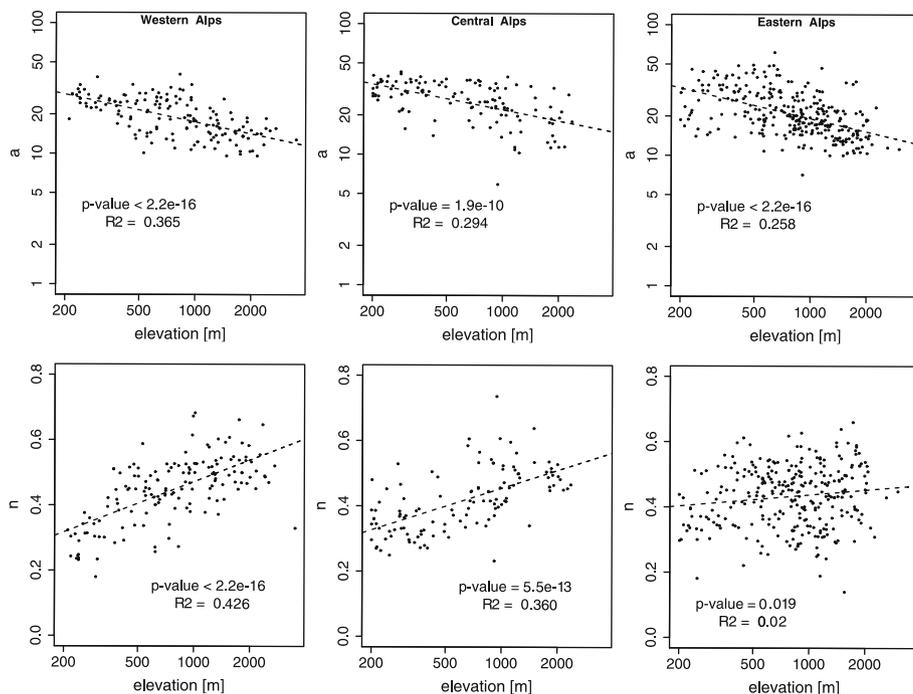


Fig. 4. Dependence of the DDF curve parameters, a and n , on elevation for the three sub-regions: western, central and eastern Alps.

Table 1

Coefficients of the model $h_d(z,x)$ estimated by linear regression for the different durations (and correspondent p -values).

	1 h	3 h	6 h	12 h	24 h
a'_0	97.3 (2.2×10^{-16})	8.5 (0.0006)	10.7 (0.0001)	9.8 (0.0001)	7.8 (0.001)
a'_1	-0.25 (0.001)	0.21 (0.02)	0.23 (0.01)	0.25 (0.004)	0.34 (0.0003)
n'_0	0.71 (0.19)	1.62 (0.01)	1.69 (0.006)	3.09 (2.2×10^{-7})	3.47 (4.9×10^{-8})
n'_1	-0.08 (0.34)	-0.25 (0.007)	-0.26 (0.004)	-0.44 (9.1×10^{-7})	-0.51 (1.4×10^{-7})

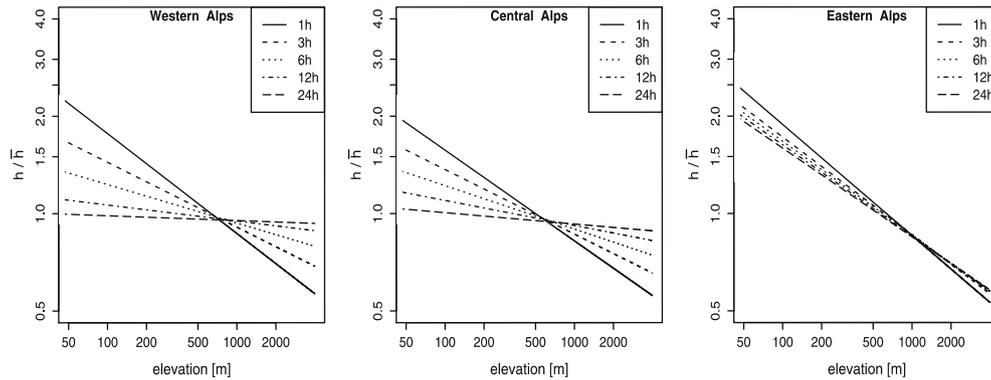


Fig. 5. Regressions between average maximum rainfall of duration d , h_d , and elevation for the three sub-regions: western, central and eastern Alps.

that shows how the coefficients a'_0 , a'_1 , n'_0 and n'_1 of Eq. (14) are univocally defined from the regressions parameters a_0 , a_1 , n_{01} , n_{02} , n_{11} and n_{12} of Eqs. (2) and (7). Alternatively, the model in Eq. (14) can be estimated by linear regression for the different durations, obtaining the results in Table 1: as expected, the dependence on elevation and longitude is again found to be statistically significant (the longitude is not significant only for the 1-hour duration). The dependence of h_d on both z and x is represented in Fig. 5 where, by fixing the longitude x , the model (14) is estimated in the three sub-regions (on the y -axis the precipitation values are divided by their average). A fan-shaped group of lines is obtained for the western and central Alps, while for the eastern region the slopes are all very similar for the different durations. This behavior reveals the existence of possible different rates and amounts of wet air arrival in this part of the study region (see e.g., Rudari et al., 2005).

5. Spatial interpolation

Polynomial and spline interpolation algorithms have been used in the past for mapping rainfall fields (Tabios and Salas, 1985), also in mountainous regions (Creutin and Obled, 1982). In such regions, however, it has been demonstrated that better results can be obtained by applying the kriging techniques (Hevesi et al., 1992). Kriging is a geostatistical method to interpolate the values of a random field at unobserved locations from observations at nearby locations. The quality of its performance depends on the size of the sample of observations (Briggs and Cogley, 1996), so that if the sample is small it may result inadequate to represent the complexity of the rainfall field (Bacchi and Kottagoda, 1995). Using a densely sampled external (or auxiliary) variable in addition to the variable of interest may help to compensate the scarcity of measured points (Gotway and Hartford, 1996) and to capture the spatial heterogeneity of the process. Modified version of the ordinary kriging technique, such as kriging with an external drift (eg., Goovaerts, 2000; Rivoirard, 2002), kriging combined with a linear regression (Knotters et al., 1995) or detrended kriging (Chua and Bras, 1982) have then been proposed in the literature. Basically in these methods a linear regression is fitted between the target variable and the auxiliary variable and then an ordinary kriging

is applied to the residuals, under the assumption that they are uncorrelated.

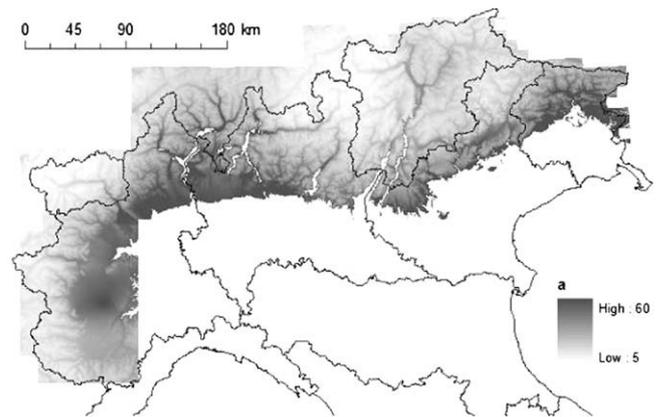


Fig. 6. Variability of the DDF parameter a over the region of study.

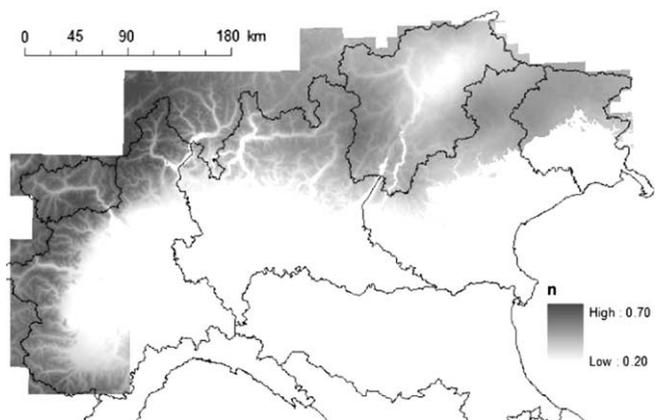


Fig. 7. Variability of the DDF parameter n over the region of study.

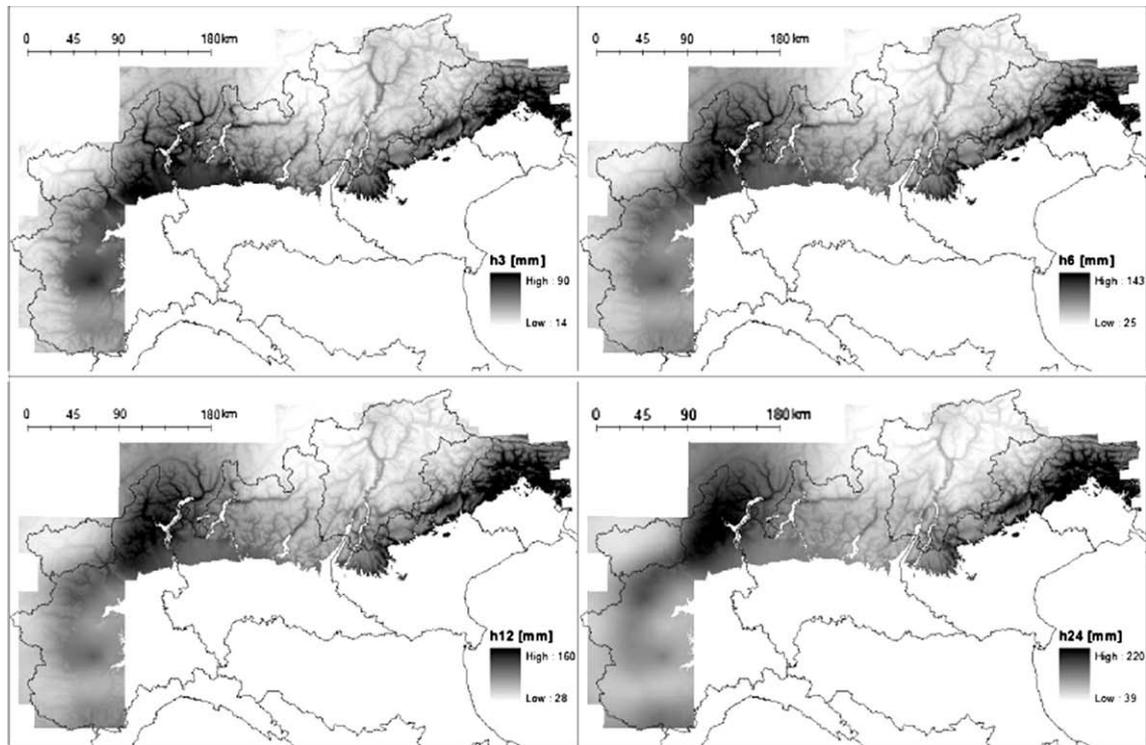


Fig. 8. Variability of h_3 , h_6 , h_{12} and h_{24} over the region of study.

In the present study the variables to be interpolated are the parameters of the depth–duration curve a and n . The spatial interpolation of the DDF coefficients, in fact, is the standard method to estimate design rainfall at ungauged sites. In practice, since the relations configured in the previous sections between the coefficients of the DDF, elevation and longitude are found to be representative of the study area, a modified version of the detrended kriging is used. Before applying the kriging algorithm, the at site values of a_j and n_j estimated for each station are recomputed as if referred to an elevation of 200 m, obtaining

$$\ln a_{200,j} = \ln a_j - a_1 \ln(z_j/200) \quad (16)$$

and

$$n_{200,j} = n_j - (n_{11} + n_{12}x_j) \ln(z_j/200). \quad (17)$$

An ordinary kriging, respectively, with exponential and pentaspherical variograms, is then applied to spatially interpolate $a_{200,j}$ and $n_{200,j}$, obtaining a representation of how these coefficients would vary over a hypothetical 200 m high flat area. The real values (i.e., accounting for elevation and longitude effects) of the coefficients are then obtained as

$$\ln a_i = \ln a_{200,i} + a_1 \ln(z_i/200) \quad (18)$$

and

$$n_i = n_{200,i} + (n_{11} + n_{12}x_i) \ln(z_i/200). \quad (19)$$

where z_i is the elevation of the i th cell of a digital terrain model with a $250 \times 250 \text{ m}^2$ grid, and x_i is the longitude of the centroid of the cell. A representation of the variability of a_i and n_i over the region is given in Figs. 6 and 7. The maps of h_3 , h_6 , h_{12} and h_{24} are shown in Fig. 8 (where the map for h_1 is not shown, being very similar to the one shown in Fig. 6).

The difference between this procedure and the standard detrended kriging is that $\ln z$ is used instead of z both in expression (16) and (17). More importantly, the relation used to detrend n in-

volves also the longitude x as an independent variable, which would not be possible with a standard detrended kriging.

6. Conclusions

The statistical relationship between intense sub-daily precipitations and elevation has been investigated for a database of 567 stations located in the alpine region of Italy. Contrary to expectations, maximum annual precipitations of short duration are found to significantly decrease with elevation. This tendency also appears to have a geographic drift from the western to the eastern side of the alpine chain.

A possible reason behind the decrease of extreme rainfall with elevation is gauge undercatch at high-elevation sites (e.g., Hamon, 1973; Golubev, 1985; Adam et al., 2006). However, the negative slope of the depth–elevation regression curves is found to decrease when considering events of longer duration. This in our opinion proves that undercatch cannot represent the dominant mechanism behind our empirical findings.

As an alternative explanation, these phenomena may be the result of the reduction with altitude of the effective condensation rate of an air mass when subjected to orographic uplift, also known as Clausius–Clapeyron effect (see e.g., Alpert, 1986; Roe, 2005). On the other hand, more efforts are required to explain the different behavior of rainstorms of longer duration between western and eastern Alps. The authors who have investigated the different meteorological mechanisms that lead to the formation of extreme precipitation across northern Italy, in fact, do not explicitly provide useful indications in this sense (see e.g. Rotunno and Ferretti, 2001; Rudari et al., 2005).

The second part of the study proposes a simple linear model that can represent the variation of the coefficients a and n of the DDF curve with elevation, where longitude is also introduced as an auxiliary explicative variable. This simple statistical tool proves to be very effective in the estimation of a design rainfall at an ungauged site, or in ungauged areas.

Acknowledgements

The work has been supported by the Italian Ministry of Education through the Grant #2006089189. The valuable help of E. Bartolini and the comments from two anonymous reviewers are gratefully acknowledged.

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