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Dimensional analysis of literature formulas to estimate the characteristic flood response time in ungauged basins: A velocity-based approach

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ABSTRACT

When investigating the wide literature dealing with the assessment of the critical time scale for basin hydrological response, several aspects still to be clarified can be acknowledged. Despite the high sensitivity of design flood peaks to the estimated time parameter value, there is still no agreement on the conceptual and operational definitions of the basin response time, resulting in several different approaches and formulations available.

In our work, we suggest a conceptual approach to either reject or recommend formulas of the characteristic basin response time for ungauged basins, with the aim of devising some practical steps in the choice of a robust formulation to be used in hydrological modelling and flood hydrograph design. To this end, 29 empirical and semi-empirical formulas, all containing a basin length and slope, have been carefully selected and their structure compared in dimensional terms, using a simple hydraulic reasoning (e.g., the Chezy formula) as indicator of hydraulic consistency. 13 *hydraulically consistent* formulas have been identified.

Starting from wave celerities and using the river network morphology of 135 watersheds in north-western Italy, we have then investigated and compared the variability of the average flow velocities estimated using all formulas whose input data are available within the study area. By comparing the magnitude and basin scale dependence of the inferred velocities with the values observed in the literature, which generally increase with basin size, some formulas are considered not reasonable, while 5 of them are identified as more robust, i.e. consistent with the observations. These are the formulas of Chow (1962), NERC (1975), SCS (1954), McEnroe and Zhao (1999) and Watt and Chow (1985).

Our findings lead to identify analytically the relationships between the exponents of each formula and those of the scaling law linking the length and slope of the basin. These relationships, driving the increase or decrease in the velocity values with basin size, allow us to identify the range of length and slope exponents in the characteristic time formulas for which velocity increases with basin area, as literature suggests. Based on the same relationships, one of the 5 formulas above can be adopted in practical applications and a guideline for calibrating new formulations can be followed.

1. Introduction

Design flood peaks for a given return period need to be estimated for hydraulic design purposes (e.g. Brunner et al., 2016).

Observed streamflow data may in some cases be inadequate to directly estimate design floods or even lacking in ungauged basins. In the last decades, runoff prediction in ungauged basins (PUB) has received great attention from hydrologists (Blöschl et al., 2013; Montanari et al., 2013) and several methods for predicting basin response are currently available. They include regionalization techniques, empirical approaches and hydrological models (see e.g. Hrachowitz et al. (2013) for a review). Traditionally, where no in-situ runoff data exist, the indirect estimation through the rainfall-runoff transformation has been used (Singh et al., 2014). A broad class of rainfall-runoff models is based on the concept of basin response function (Sherman, 1932), which can be adopted to obtain discharges at the basin outlet produced by any given excess design rainfall through convolution. One of the key parameters that may be needed to specify the response function is the characteristic time of the basin hydrological response.

It is well known that the estimation of this characteristic flood response time is a difficult task, to which design flood estimates are

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Nomenclature

A	Catchment area.
A_{imp}	Fraction of impervious area.
c	Celerity.
C	Chezy coefficient.
$clc1$	Corine land cover classification 1. Percentage, on the basin area, of continuous and discontinuous urbanized areas (Corine classes 111, 112). Corine classes are available at https://land.copernicus.eu/.
$clc4$	Corine land cover classification 4. Percentage, on the basin area, of non-vegetated areas (Corine classes 331, 333, 332, 334), mining areas, landfills, and construction sites (Corine classes 131, 133), industrial and commercial areas, communication networks (Corine classes 121, 122, 123, 124). Corine classes are available at https://land.copernicus.eu/.
CN	SCS Curve Number.
CN_S	Soil storage coefficient of the SCS-CN method.
E_f	Channelization factor in Espey et al. (1966) formula.
H_{max}	Basin maximum elevation.
H_{min}	Basin minimum elevation.
$H_{max,LDP}$	Basin elevation at the ending point of the longest drainage path (on the drainage divide).
$H_{min,LDP}$	Basin elevation at the starting point of the longest drainage path (on the outlet).
i	Net rainfall intensity.
i_2	Two-years return period rainfall intensity.
L_c	Length of the main channel (distance from the outlet to the end point of the visible stream).
L_{ca}	Length of the main channel up to the point on the stream closest to the centroid of the basin.
L_{LDP}	Length of the longest drainage path (distance from the outlet to the drainage divide).
n	Roughness coefficient.
q_{wm}	Weighted mean discharge in Askew (1970) formula, defined as "*the mean rate of total discharge over the time of occurrence of direct runoff, weighted in proportion to the direct runoff discharged at that rate*".
R	Hydraulic radius.
RD	Road density, computed as the total length of roads and streets in the watershed, divided by the drainage area.
R_e	Basin elongation ratio (ratio of the diameter of a circle of the same area as the basin to the longest drainage path length).
R_s	Basin shape factor (ratio of the watershed area to the main channel length squared).
S_b	Average basin slope, computed by averaging the slope map obtained from a digital elevation model (if not otherwise specified).
S_c	Average slope of the main channel (details are provided in the Supplementary Material, section S.1).
$S_{diff,LDP}$	Ratio of the fall in elevation of the longest drainage path between the divide and the gauging station to the length of the longest drainage path.
S_{LDP}	Average slope of the longest drainage path (details are provided in the Supplementary Material, section S.1).
$S_{c,w-L}$	Weighted average slope of the main channel, computed as suggested by Laurenson (1962).
$S_{c,w-T\&S}$	Weighted average slope of the main channel, computed as suggested by Taylor and Schwarz (1952).
$S_{10-85,LDP}$	Average slope of the drainage path computed as the elevation between two points on the channel (extended to the drainage divide), located 10 and 85 % of the channel length from the outlet, divided by the length of the channel between the two points (Benson, 1959).
t_L	Lag time – Computed as the time difference between the centroids of net rainfall and hydrograph.
t_p	Lag time – Computed as the time difference between the centroid of net rainfall and the hydrograph peak.
t_{peak}	Time to peak.
t_c	Time of concentration. Estimation procedures for each formula are given in Table A.2 of Appendix A.
W	Storage factor in Kennedy and Watt (1967) formula.
δ	Channel shape factor.
$a, a_1, d, k_1, k_2, k_3, m_1, \gamma, \gamma_1$	Constants.

however highly sensitive. Already Bondelid et al. (1982) showed that about 75 % of the total error in estimating flood peaks could be attributed to errors in response time estimation. In Gericke (2018) it emerges how under or overestimating the response time can have a considerable impact on the design flood and thus on the design of hydraulic infrastructures.

A wide literature dealing with the identification of the time parameter of the basin hydrological response during flood conditions exists worldwide for both gauged (Bell and Om Kar,1969; Gericke and Smithers, 2014) and ungauged (Michailidi et al., 2018; Nagy et al., 2021) watersheds. In this regard, it can be useful to refer to the theories underlying the evaluation of the basin IUH through geomorphology (the GIUH theory, Rodriguez-Iturbe and Valdes (1979) or Gupta et al. (1980)) and some further efforts to parametrize the IUH (Rosso, 1984). In all these cases, the time-scale parameter estimation issue has remained largely unsolved. Correspondingly, to date one may admit that there is no general agreement on which parameter is best suited to summarize the hydrological flood response of a watershed and thus to allow the synthesis of a design flood hydrograph. The same divergency is experienced when dealing with the different conceptual approaches and operational computation methods that have been proposed over the years to estimate the same time parameter (e.g. McCuen, 2009).

In some uncommon cases, direct estimation of travel times along the river network was undertaken, e.g. tracer measurements. In a study performed in Australia, Pilgrim (1977) adopted the tracer's travel time during flood runoff, i.e. the time from the injection of the tracer to the center of mass of the tracer's activity curve, as a measure for the basin response time. Later on Azizian (2019), although not under flood conditions, estimated travel times in seven small sub-basins of the Meime watershed (Iran) using a salt tracer, measuring the delay between the entry of salt in the upstream river reach and the time of the peak electrical conductivity in the downstream one.

Most commonly, when observations are available, direct assessments are made by using observed rainfall and runoff time features (e.g. Chow et al., 1988). The peak time of direct runoff, the times of centers of mass of net rainfall and direct runoff, the peak time of net rainfall and the time of the end of net rainfall are commonly adopted as time indicators. Time distances between pairs of them have been used to compute, in practice, the most frequently adopted travel time parameters: the lag time, the time of concentration and the time to peak (the reader can refer to McCuen (2009) for a detailed description of these parameters). It must be specified that the time to equilibrium, i.e. the time from the start of a constant rainfall to the time when inflow equals outflow, is not dealt with in this paper, because it is often assumed to be equal to the time of concentration (McCuen, 2009). While we are aware of the arguments raised by Beven (2020) about the difference between these two parameters, they are not addressed here as it goes beyond the scope of this work.

Likewise, this paper will not discuss the above mentioned different operational methods to calculate these time parameters. However, it is worth saying that at least six different working definitions for the time of concentration and seven for the lag time have been proposed in the literature (see e.g. McCuen, 2009; Espey et al., 1966). Moreover, some of them overlap, contributing to confusion in design practice for hydrological applications.

When attributing any meaning to the characteristic time being considered, one must bear in mind that, as written by Leopold (1991), response time "is a fingerprint of the drainage basin, reflecting the storage and *velocity of water* in its travel over the basin and down channel". However, it is not the *water flow velocity*, which is referred to by tracer measurements, but rather the *flood wave celerity*, which is more commonly referred to by hydrograph analysis methods, which should be considered when addressing the response time estimation. As Beven (2020) wrote, "*if we are interested in hydrograph responses, it should be clear that we should be not so much concerned with the time of travel of an input water particle from the farthest reaches of the catchment to the outlet as with the time it takes for the effect of that input to have an effect on the output*". Although this aspect is receiving increased attention in recent years (e.g. McDonnell and Beven, 2014; Beven, 2020; Shook et al., 2023), it is not mentioned at all in past works.

All the above cited direct methods typically provide a wide range of time values for the same basin and many studies have indeed indicated that the response time should vary with the flow rate, rainfall intensity, antecedent soil moisture condition and associated factors. For this reason, citing Efstratiadis et al., (2014), "*finding the basin constant*" response time "*is an enigma*".

Some simplifying assumptions are needed to overcome complexities, both because a single time parameter is frequently required for design purposes and because less information is available in ungauged basins, to which this work is targeted. These simplification are applied by neglecting the less influencing and more difficult to calculate parameters controlling the various phenomena that compose basin response.

This pragmatic attempt towards simplification is shaped by the use of existing formulas, suggested for design purposes in ungauged basins. This is what motivated, from the professional practice perspective, the development of the present study since hydrologists, when facing the estimation of flood peaks in ungauged sites, do not have much guidance in choosing the most suitable response time formulation for their purposes.

Empirical formulas are generally the product of regression analyses between the observed watershed response times and morphological, land use or event parameters, indirectly taking into account physical processes occurring in the watershed. On the other hand, also theoretically based equations exist. They generally account for theoretical principles in an explicit form, but typically involve a larger number of input parameters.

A huge number of formulas is available in the literature: the works by Carter (1961), Chow (1962), Kirpich (1940), McCuen et al. (1984), and SCS (1954) are among the most cited. Some of these formulas, more than others, have become popular over the years. However, this popularity does not always correspond to estimates whose uncertainty is clearly understood. At the same time, because multiple definitions with different assumptions have been coined over the years, formulas have been often misused. It must also be acknowledged that formulas to compute the same time parameter can provide very different estimates. Grimaldi et al. (2012), for instance, showed that time estimates can vary by up to a factor of 5. Hence the choice of one formula over a different one can become crucial for the final design flood estimates.

This work is an attempt to overcome the fragmentation problem outlined above and to select a limited pool of formulas providing *hydraulically consistent* and robust results to be used in the prediction step of the hydrological modelling.

Part of this work will focus on drawing some order among 29 empirical and analytical formulas carefully selected from the literature. To this end, we did not use review papers already available in the literature, but the original papers where formulas are published have been referred to. We also clearly report the suggested range of application, the units of measurement of the input variables and, in some cases, the actual meaning of the time parameter being estimated. The main novelty of this work, however, is the comparative assessment of formulas, after their classification on the basis of a simple hydraulic reasoning, through application in 135 basins in North-Western Italy, considering the inferred spatio-temporal average velocities at basin scale that each formula produces. The term "inferred" is used to indicate that average velocities are derived from flood wave celerities associated to the formulas.

The average velocity is used here as a more meaningful parameter for comparisons than travel time. Travel times cannot easily be compared with reference values because their scaling with lengths and basin areas is quite undefined. In other words, it is difficult to identify the "object" for which we measure travel time and its "true" value cannot be directly measured or determined (Sharifi and Hosseini, 2011). This uncertainty can be partially solved when relying on the inferred velocity estimates that each formula provides.

One might question the need to compare the characteristic times provided by formulas with observed ones. However, this comparison may lack robustness if considering the high subjectivity that the above-described estimation methods from rainfall and runoff data are affected by. Furthermore, there are already plenty of review papers where this approach is adopted with the aim, on the one hand, to quantify the variability (and uncertainty) of response times provided by each formula for a given basin (e.g. Azizian, 2018) and, on the other hand, to verify their accuracy in reproducing "observed" times (e.g. Ravazzani et al., 2019).

From a scientific standpoint, more than one reason led to the conception of this study. Unlike previous studies, we will not judge the formulas according to their ability to reproduce observed travel time parameters. Instead, we will calculate implied velocities from wave celerities, and assess how velocity varies with catchment size, and whether this is consistent with observations. To the best of the authors' knowledge, this is a new approach in the available literature. The only, very recent, example where velocities produced by literature formulas are investigated is the work of Shook et al. (2023). However, the approach adopted there is the one common in the literature of assessing the performance of each formulation in reproducing observed times and, in this case, stream velocities. The goal of the work is also quite far from ours, since in Shook et al. (2023) the authors investigate the validity of the common literature formulas when applied to the context of the Canadian prairies. Our goal is more generic, i.e. to investigate which of the formulas produces implied velocity estimates whose general behavior is consistent with observations.

Finally, a further contribution of this work lies in identifying analytical relationships between velocities and basin morphological features, to be used as helpful tools when calibrating new formulas.

The paper is organized as follows: in section 2, each formula is carefully documented, as well as the data used to develop it, the units and the range of derivation. Using hydraulic reasoning, the formulas are then grouped based on their mathematical structure.

After presenting the study watersheds in section 3, formulas are applied on 135 basins in north-western Italy to check their behavior in section 4. This is done by analysing the average spatio-temporal velocities within the basins. In section 5, results are discussed, and some recommendations and conclusions are drawn in section 6.

2. Formulas for the estimation of characteristic response times

2.1. Formulas consistency and background

As already mentioned in the Introduction, when estimating the hydrograph time parameters in ungauged basins the user has to choose from a large set of available relationships and must face some difficulties in their practical use. This evidence has already been discussed in some papers, among which the clearest problem statement is due to Sheridan (1994), who wrote: "*Several factors may contribute to the current confusion regarding the choice of appropriate procedures for estimating watershed hydrograph time parameters […] Some of the contributing factors are: (i) Differing definitions and lack of consistency in parameter definition (ii) Lack of consistent parameter input and units (iii) Development of empirical relations within a limited physiographic region*".

Among the issues listed by Sheridan (1994), the use of proper input parameters, i.e. those used when calibrating the formula, is a critical aspect. Let's take the example of the basin slope. Among the 29 formulas reviewed in the following, the term "slope" could refer to basin or channel slope, and even the channel slope is defined in different ways, not always clearly recognizable in the literature, so that the user's choice of the correct slope parameter becomes critical. Errors in selecting the appropriate input parameters and units are frequent and, as it will be shown later, may have significant impacts on the estimated response time.

The 29 formulas considered in this work have been carefully selected from review papers (Schulz and Lopez, 1974; Singh, 1988; Sheridan, 1994; Gericke and Smithers, 2014; Azizian, 2018; Michailidi et al., 2018; Ravazzani et al., 2019). Rossi's formula (Rossi, 1974) has been also added to the pool. The list of formulas, as well as units and the input parameters' meaning, is given in Table A.1 of Appendix A. Not all the formulas available in the review papers accessed have been included in this list, because some criteria have been adopted to select any of the 29 formulas, i.e.:

1. The formula includes a measure of length and slope. According to our aim to seek parsimony, it has been assumed that an expression relating response time to watershed physical features should be relatively simple and related to parameters that must be easily determined, e.g. from digital elevation models.
2. The original publication is available. It is worth clarifying that the above cited review papers have been only used to identify formulas, while the original articles have been referred to for the characterization of each formula;
3. The time parameter is clearly defined, including units;
4. The input parameters are clearly defined, including units.

Some problems were encountered, for instance, in classifying the Jonhstone and Cross formula (1949), which has not been included here. In Johnstone and Cross (1949), the authors wrote: "*We refrain from giving the numerical values of the experimental constants*" of the formula "*because successful use of the empirical equations depends not only upon those values but upon the observance of especially prescribed techniques for measuring r, L, S, W and R, which space limitations preclude describing here*". However, Johnstone and Cross (1949) is the reference commonly cited in the literature for this equation. The Picking formula has a similar problem, and its original source could not be found. Lopes da Silveira (2005) wrote: "*With the same difficulty in knowing its origin and applicability is the Picking equation, whose reference consulted (*Pinto et al., 1976*), says nothing about it*".

Table A.2 of Appendix A includes, for each formula, the application range and the number of basins used for its calibration. The application range is expressed using the same units for all formulas. In order to provide an estimate of the popularity of each formula, its number of Google Scholar citations is also provided. Where available, Table A.2 also contains some details on the derivation of the equation, such as the fitting performance, which is expressed in terms of the coefficient of determination R^2 or the regression correlation coefficient ρ.

2.2. Mathematical classification of formulas through dimensional analysis

Once the main features of formulas have been clarified in Tables A.1 and A.2 of Appendix A, they were classified according to their mathematical structure, following a simple "hydraulic reasoning", i.e. relying on a few basic theoretical principles that can help to discriminate between "similar" or intrinsically different formulas. In the following derivations, we will use hydraulic reasoning to derive (i) time from length and celerity, (ii) celerity from velocity and (iii) velocity from hydraulics.

Based on Table A.1, we can assume that response time can be described as a function of the ratio between a characteristic length L and a characteristic slope S of the basin, raised to a given power, according to:

$$t = \gamma \frac{L^{\alpha}}{S^{\beta}} \tag{1}$$

where α, β and γ may vary for different basins and are typically determined empirically. This coupling of variables is common to most of the characteristic time formulations of any type found in the literature (and thus included in this work). In other literature formulas, length can be replaced by basin area (e.g. Dooge, 1973) or slope by difference between the maximum elevation of the whole catchment and the outlet section (e.g. California Highways and Public Works, 1955). Other rather common formulas, on the other hand, include only one parameter, which may be the basin area (e.g. Pilgrim and McDermott, 1982; Flavell, 1983) or the main channel length (e.g. Haktanir and Sezen, 1990).

As already mentioned, we are interested in estimating inferred velocities from each formula and checking their reasonableness, i.e. checking whether they are consistent with observations. According to what discussed in the Introduction, the basin response time can be expressed as:

$$t = \frac{L}{c} \tag{2}$$

where c is the wave celerity, i.e. the speed at which perturbations to the flow propagate through the system. In order to compare the velocity values resulting from the application of each formula and the observed ones, moving from celerities to velocities is required. Following the derivation from the kinematic wave theory and the simplifications adopted by Lighthill and Whitham (1955), celerity c is linearly related to flow velocity v :

$$c = k_1 v \tag{3}$$

where the k_1 value can be estimated using either the Manning (Manning, 1889) or Chezy (Chezy, 1775) resistance equations. For wide rectangular, triangular or parabolic channels, the k_1 values will be, respectively, 1.67, 1.33, and 1.44 using the Manning equation, and, respectively, 1.50, 1.25, and 1.33 using the Chezy equation (Sriwongsitanon et al., 1998). Adopting the simplest possible configuration, the hypothesis of the validity of Chezy's law in a wide rectangular channel, k_1=1.5 is assumed in this work.

According to Chezy (Chezy, 1775) and under the assumption of uniform flow, the average streamflow velocity v can be expressed as:

$$v = C\sqrt{RS} \tag{4}$$

where C is the Chezy coefficient, R is the hydraulic radius and S is the drainage path slope. Taking C and R as constants, v can be considered to be roughly proportional to the square root of the drainage path slope:

$$v = k_2\sqrt{S} \tag{5}$$

By coupling Eqs. (2), (3) and (5), it then follows:

$$t = \frac{1}{1.5k_2}\frac{L}{\sqrt{S}} = \frac{\gamma}{1.5}\frac{L}{\sqrt{S}} \tag{6}$$

The α and β exponents of Eq. (1) are therefore 1 and 0.5, respectively, if we assume the Chezy velocity formula with constant hydraulic radius and roughness. In other words, Eq. (6) is a specific case of the more general Eq. (1).

The average velocity v can be also expressed using other hydraulic formulas, e.g. the Gauckler-Manning's formulation (Manning, 1889), which provides:

$$v = \frac{1}{n}\sqrt[3]{R^2}\sqrt{S} \tag{7}$$

where n is the Gauckler-Manning coefficient. By adopting the same assumptions and substitutions as above, similarly the exponents of L and S are 1 and 0.5.

As an alternative to empirical formulas like the one in Eq. (1), the kinematic wave model can be adopted to provide an estimate of response time. The model can be generalized as follows (McCuen and Spiess, 1995):

$$t = \frac{\gamma_1}{i(t)^{\nu}}\left(\frac{nL}{\sqrt{S}}\right)^{\mu} \tag{8}$$

where $i(t)$ is the net rainfall rainfall intensity with duration equal to the time of concentration, while μ, ν and γ_1 are coefficients. γ_1 depends on the units system used, while μ and ν depends on the assumptions being made. Assuming the Manning's equation leads to$\mu = 0.6$ and$\nu = 0.4$ (e.g. Woolhiser and Liggett, 1967). The Soil Conservation Service (Welle and Woodward, 1986) proposes a simplified solution that does not require an iterative procedure for $i(t)$, replacing the rainfall intensity with duration equal to the time of concentration with the 2-year, 24-hr rainfall. In this case the coefficients values are$\mu = 0.8$ and$\nu = 0.5$.

Regardless of the specific values assigned to the coefficients, the structure of Eq. (8) again indicates that the ratio between the length and slope exponents is 2:1.

Hydraulic formulas thus suggest that response time can be expressed as a function of the *basin factor* $L/S^{0.5}$. In other words, the β exponent in Eq. (1) is equal to $\frac{\alpha}{2}$ under specific assumptions. Formulas in which this ratio is satisfied will henceforth be referred to as *'hydraulically consistent'*. This arrangement occurs frequently among empirical formulas (e.g. Kirpich, 1940; Carter, 1961; Chow, 1962; McEnroe and Zhao, 1999; NERC, 1975; Pezzoli, 1970; Rossi, 1974) as depicted in Fig. 1, where each circle represents a formula. The closer the circle is to the line, the closer is the formula to the theoretical ratio discussed above.

A total of 13 *hydraulically consistent* formulas were identified, i.e. those of Carter (ID = 5), Chow (ID = 6), Kennedy and Watt (ID = 9), Kirpich (ID = 10), McEnroe and Zhao (IDs = 13, 14, 15), NERC (ID = 16), Overton and Meadows (ID = 17), Pezzoli (ID = 19), Putnam (ID = 20), Rossi (ID = 21) and Watt and Chow (ID = 28).

It should be noted that the Linsley et al. (ID = 11), Schulz (ID = 23) and Sheridan (ID = 25) formulas are not considered to be *hydraulically consistent*, as their numerator is the product of two length parameters, i.e. $(L_{ca}\bullet L_c)$ or $(L_{ca}\bullet L_{LDP})$, thus the ratio between the α and β exponents is not 2:1, but 4:1.

3. Data and features of application of formulas

3.1. Study area and available data

For the purposes of this paper, an extended synthetic sample of typical watersheds could have been produced, allowing to perform a sensitivity analysis of the formulas from a theoretical point of view. However, referring to real watersheds, and therefore to actual combinations of basin features (particularly of length and slope) is certainly important when providing practical directions on the use of available formulas.

To this end, a real study area included in a region of about 25000 km^2 has been considered. The study area consists in 135 basins in north-western Italy, as shown in Fig. 2. Their areas range from 3.5 to 8020 km^2, except of three Po River sub-basins, which have an area larger than 10000 km^2 (red outlets in Fig. 2a). Empirical cumulative distributions of

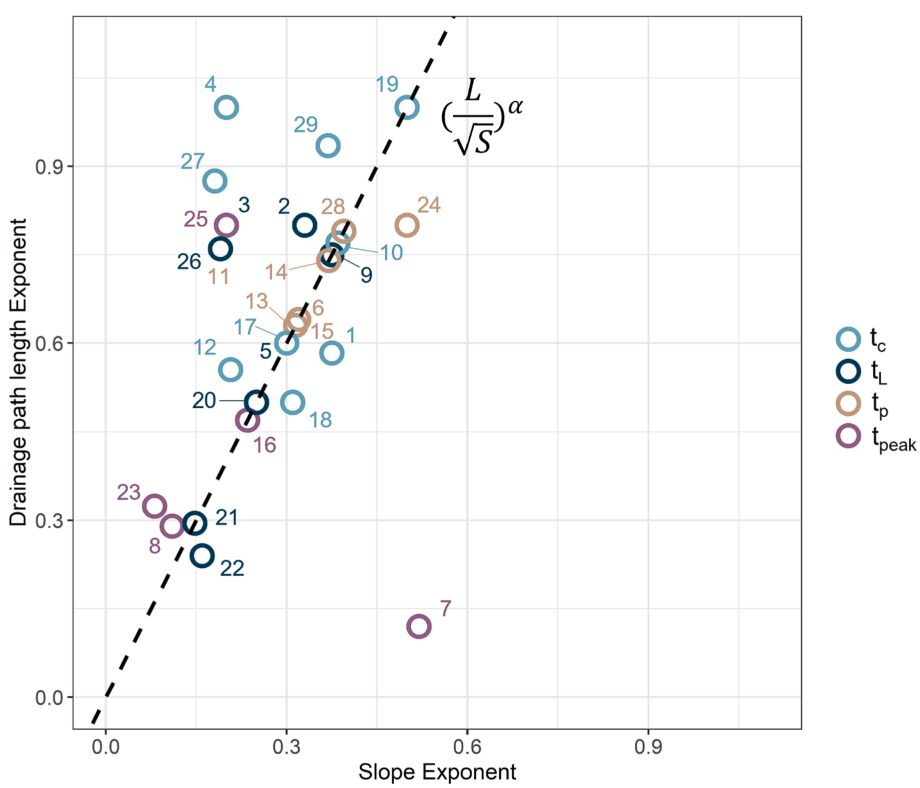

Fig. 1. Plotting of formulas on the plane of Length-Slope exponents. Numbers refer to the formula IDs given in Table A1.1. The dashed line has a slope of 2:1 between the exponents α and β.

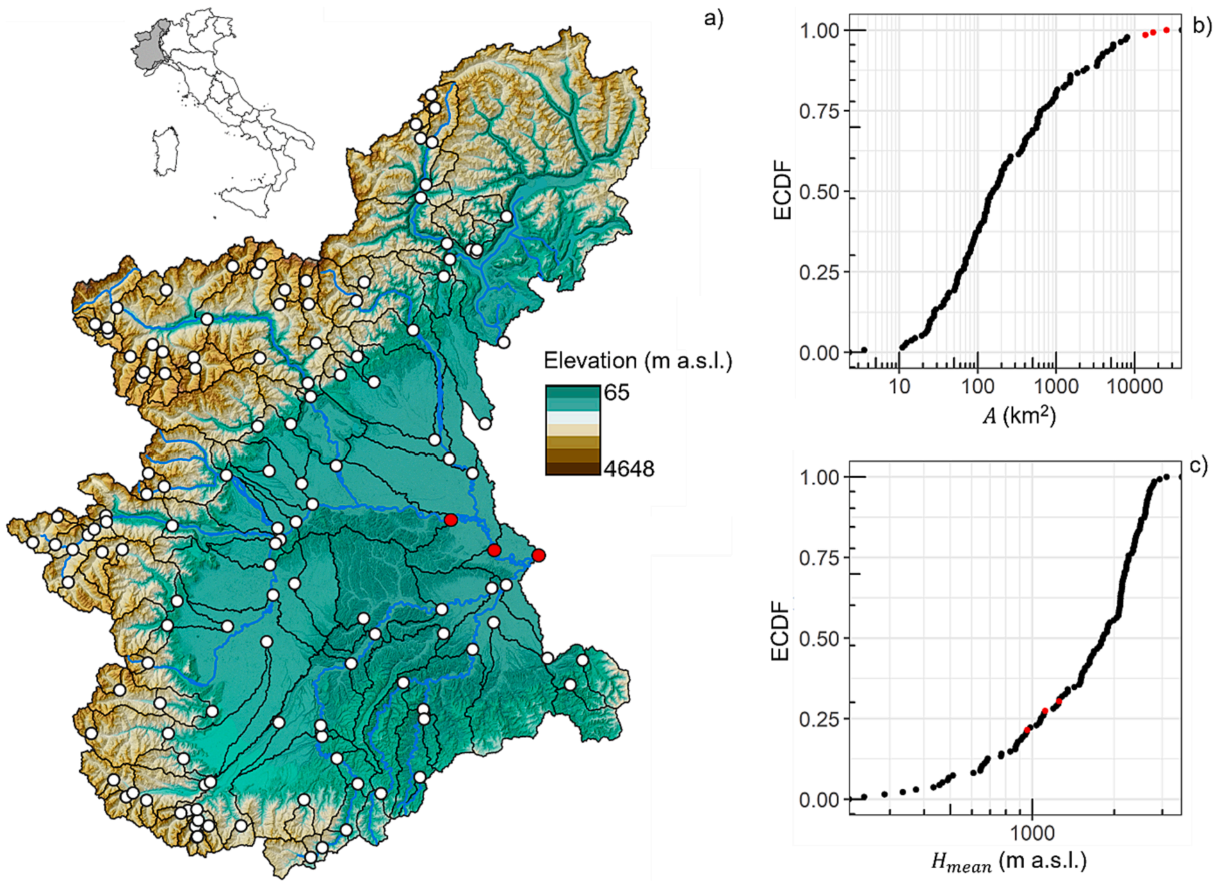

Fig. 2. (a) Location and Digital Elevation Model of the study area. Red outlets refer to Po river sub-basins with an area larger than 10000 km^2. Moving upstream: Po at Isola Sant'Antonio ($A = 25597$ km^2), Po at Valenza ($A = 17226$ km^2), Po at Casale Monferrato ($A = 13689$ km^2). (b) and (c) Empirical cumulative distribution functions (ECDF) for basin areas (panel b) and mean elevations (panel c) for the 135 basins. The ECDF is defined as $\frac{i}{N}$ for $i = 1, \ldots, N$ where i is the ordered variable for each watershed. Red dots in panels b and c refer again to the three Po river sub-basins. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

basin areas and mean elevations are provided in Fig. 2b and 2c, respectively. Mostly small and medium–high elevation watersheds are considered in this study, as about 75 % of them have a mean elevation higher than 1000 m a.s.l.

In order to investigate how formulas perform when applied to a large set of basins with different characteristics, morphological and land use features were computed. A summary of the available data for each basin is provided in Table 1. While the meaning of the length parameters considered in this work is clearly defined, different definitions exist for the basin's representative slope. A detailed description on how each parameter have been computed can be found in the Supplementary Material (section S.1).

With the aim of providing generality to the results presented in this work, section S.1 of the Supplementary Material also provides an assessment on how other geographical areas can be well represented by the topography of north-western Italy.

Table 1
Available data. Units, quantiles and range of variability for each feature are provided. For symbols, the reader can refer to the Nomenclature.

Parameter sub-category	Parameter	u.d.m.	Quantile 25 %	Quantile 50 %	Quantile 75 %	Range of variability
Altimetrical and geometrical	A	km^2	64	159	662	3–25597
	H_{max}	m a.s.l.	2633	3222	3756	418–4792
	H_{min}	m a.s.l.	230	625	1265	27–2697
	$H_{max,LDP}$	m a.s.l.	2512	3006	3207	336–4284
	$H_{min,LDP}$	m a.s.l.	243	637	1307	68–2697
	L_{ca}	km	5.6	10.8	30.5	0.3–158.8
	S_b	%	21	25.8	28.8	4–35.2
	L_c	km	12.3	25.2	69.3	2.3–309.6
Streamflow network	L_{LDP}	km	14.4	27.4	71.4	3.1–311.7
	S_c	m/m	0.027	0.063	0.104	0.011–0.248
	S_{LDP}	m/m	0.044	0.073	0.101	0.02–0.169
	$S_{diff,LDP}$	m/m	0.028	0.071	0.121	0.007–0.248
	$S_{10-85,LDP}$	m/m	0.012	0.044	0.107	0.002–0.291
Watershed shape features	R_e	–	0.47	0.59	0.71	0.26–4.74
Soil and Land use	CN	–	57.3	62.2	65.6	40.5–74.6
	$clc1$	%	0	0.7	2.5	0–12.1
	$clc4$	%	0	0	0	0–0.1

With the available data, 17 formulas could be applied, as will be specified in the following, and three different definitions for the length parameter (L_c, L_{LDP} and L_{ca}) and five for the slope parameter ($S_c, S_{LDP}, S_{10-85,LDP}, S_{diff,LDP}$ and S_b) have been used. In the Supplementary Material (section S.2) scatter plots between each of the length and slope parameters considered in this work are shown (Figures from S2.1 to S2.11), as well as the power law equations that best approximate their relationships. Scatter plots also show relationships between length and slope parameters and the basin areas (Figures from S2.12 to S2.18). The L_{ca} length parameter, which only appears as products such as $(L_c \bullet L_{ca})$ or $(L_{LDP} \bullet L_{ca})$ (formulas with IDs = 11, 23, 25), is not investigated in this work.

It is worth stressing that choosing the correct input parameters is an essential step for an accurate use of formulas. A sensitivity analysis of the response time estimates to the input parameters, in particular the slope parameter, is given in Appendix B. It is worth noting that this issue relates more to the accurate application of the formulas, i.e. the way the formulas were designed, and does not compromise the robustness of the method proposed here as a tool for appraising formulas reliability.

3.2. Rationale for the application of formulas

In order to identify which formulas are more robust, the average streamflow velocities provided by each of them have been computed over the above mentioned 135 basins in north-western Italy. According to Eqs. (2) and (3), velocity was computed as follows:

$$v = \frac{L}{1.5t} \tag{9}$$

where t is obtained from the formulas.

In section 2.2, 13 *hydraulically consistent* formulas were identified. At this stage, however, we computed velocities using all formulas for which the input parameters are available. Specifically, the following categories of formulas were not applied:

i) formulas containing the hydraulically weighted average slopes $S_{c,w-L}$ and $S_{c,w-T\&S}$, computed according to Laurenson (1962) and Taylor and Schwarz (1952), respectively.
ii) formulas containing rainfall intensity with duration equal to the time of concentration, and thus requiring iterative procedures;
iii) formulas containing parameters of complex determination, such as soil features or Manning roughness coefficient.

Summarizing, the formulas explicitly considered hereinafter are those of Chow (ID = 6), Kirpich (ID = 10), McEnroe and Zhao (1999) (ID = 13), McEnroe and Zhao (2001) (ID = 14), NERC (ID = 16), Pezzoli (ID = 19), Putnam (ID = 20) and Watt and Chow (ID = 28) within the *hydraulically consistent* group, and those of Bocchiola et al. (ID = 3), Bransby-Williams (ID = 4), Linsley et al. (ID = 11), Schulz (ID = 23), SCS (ID = 24), Sheridan (ID = 25), Temez (ID = 26) USGS (ID = 27), and Williams (ID = 29) among the *non hydraulically consistent* formulas. They are 17 formulas, though not all *hydraulically consistent*.

In particular, we will investigate:

a. the variability of the orders of magnitude of velocity values provided by each formula;
b. the pattern of velocities as a function of morphological features, i.e. the basin area and the *basin factor* $L/S^{0.5}$.

4. Assessment of inferred velocities

4.1. Standardization of time parameters for estimating velocities

When translating the time parameter estimation results to average streamflow velocities, according to Eq. (9), it must be borne in mind that not all formulas provide the same time parameter. A harmonising operation is therefore needed in order to allow the estimates to be compared. The average velocity is calculated here as the ratio between the length of the longest drainage path and, consistently, the time of concentration. It is well-known that, for a given basin, the time of concentration t_c is usually greater than the lag time t_{lag}, where t_{lag} can refer to both t_L and t_p. The relationship between these characteristic times is supported by a wide literature and the estimation of t_c, more complex, commonly relies on the estimation of t_{lag} (Gericke and Smithers, 2014). Overton and Meadows (1976) analytically found:

$$t_c = 1.6t_p \tag{10}$$

while the Soil Conservation Service (SCS, 1975)) showed that:

$$t_c = 1.67t_p \tag{11}$$

and

$$t_c = 1.417t_L \tag{12}$$

Eq.(11), also recommended by the Natural Resources Conservation System (NRCS, 2010), has been widely accepted in engineering practice for several decades. On the other hand, McCuen et al. (1984), with the aim of verifying Eq. (12), found:

$$t_c = 1.35t_L \tag{13}$$

In this work, a conversion factor of 1.67 is adopted for t_p and 1.42 for t_L. As for the time to peak t_{peak}, according to Ramser (1927), it is dealt with in this context as t_c and no conversion factor is adopted for this parameter.

4.2. Magnitude and variability of velocities according to morphological features

Based on the previous section, the equation adopted operationally to calculate v is as follows:

$$v = \frac{L_{LDP}}{1.5k_3t} \tag{14}$$

being$k_3 = 1.67$ if $t = t_p$, k_3=1.42 if $t = t_L$, k_3=1 if $t = t_c$ or $t = t_{peak}$. Only for Pezzoli's formula (ID = 19), which is not calibrated on rainfall and runoff data but analytically derived as the ratio between L_{LDP} and the Chezy's formula for average velocity, the conversion factor of 1.5 between velocity and celerity is not applied.

Velocity estimates for all 135 study watersheds as a function of the *basin factor* $L_c/S_c^{0.5}$ are shown in Fig. 3, where panel (a) refers to the hydraulically consistent formulas, while panel (b) to all others.

Each colour of Fig. 3 refers to a formula, while each dot to a watershed and a linear regression line is plotted for each of the formulas. This type of representation was chosen in order to highlight more clearly the variations between different formulas. It can be easily recognized how velocities from different formulas have different behaviors as the *basin factor* $L_c/S_c^{0.5}$ varies. The slope values of the trend lines in Fig. 3 are given in Table 2. Moreover, a large variability of typical values of velocity occurs between formulas, with values ranging from 0.1 to almost 6 m/s.

The assessment of how velocities change as the *basin factor* varies might appear as an end in itself and one may ask what is the meaning of $L/S^{0.5}$ and how it relates to basin features. The *basin factor* $L/S^{0.5}$ can be considered as a proxy for the basin area whatever specific parameters are used for L and S, as strong positive relationships can be observed (see Figures from S4.1 to S4.10 in the section S.4 of the Supplementary Material). A power law of the type:

$$A = m_1\left(\frac{L}{\sqrt{S}}\right)^{m_2} \tag{15}$$

can be written, where m_1 and m_2 are site-specific coefficients, which values depends on the particular length-slope pair considered. In

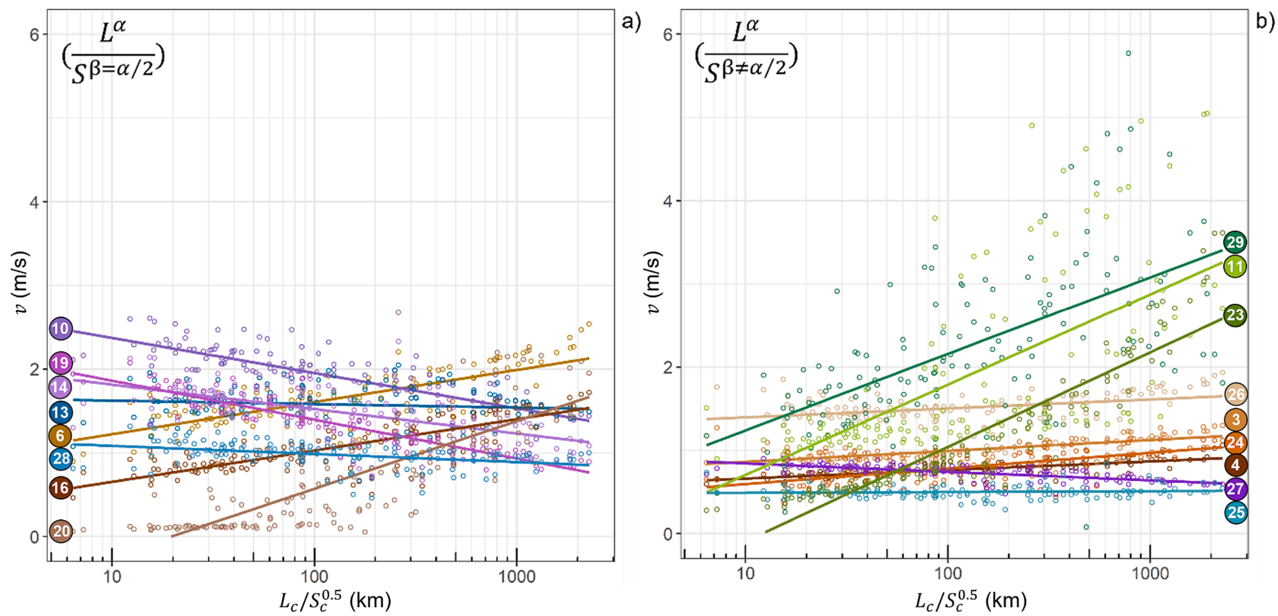

Fig. 3. Velocity estimates as a function of $L_c/S_c^{0.5}$. IDs refer to those shown in Table A.1 of Appendix A. Formulas providing similar behaviors are grouped with similar colors.

Table 2
Slopes of the regression lines identified in Fig. 3. *Hydraulically consistent* formulas are highlighted in bold text. Velocities provided by formulas with IDs = 13, 25 and 28, whose regression lines have a slope of the order of 10^{-5} (regardless of the sign), can be considered to be constant with the *basin factor*.

Formula ID	Ratio α/β	Slope of the regression line $(\frac{m}{s\cdot km})$
3	4	1.95E-04
4	5	1.38E-04
6	**2**	**5.22E-04**
10	**2**	**−4.54E-04**
11	4	1.30E-03
13	**2**	**−6.44E-05 ~ 0**
14	**2**	**−3.41E-04**
16	**2**	**4.76E-04**
19	**2**	**−4.87E-04**
20	**2**	**1.01E-03**
23	4	1.57E-03
24	1.6	2.30E-04
25	4	2.74E-05 ~ 0
26	4	2.10E-04
27	4.8	−1.17E-04
28	**2**	**−7.62E-05 ~ 0**
29	2.5	8.23E-04

Table 3, the m_1 and m_2 values for each pair *L-S* considered here are provided. For basins in north-western Italy the m_2 values range from 1 to 1.6. As a further example, Table 4 shows the m_1 and m_2 coefficients obtained when fitting the relationship in Eq. (15) to the data used to calibrate some of the formulas, i.e. those of NERC (ID = 16), Putnam (ID = 20), Temez (ID = 26) and USGS (ID = 27). Despite the different geographical regions from which data are collected, the m_2 factor varies again between 1 and 1.6.

This evidence allows us to consider, henceforth, the *basin factor* $L/S^{0.5}$ as conceptually equivalent to the basin area. This finding will become relevant in the following sections.

4.3. Consistency of velocities with observations

The first question we want to address is how reasonable, i.e. consistent with observations, the velocity estimates obtained are. Studies providing maximum velocity measurements or cross sectional mean velocities exist in the literature, where different measurement methods are used for different case studies (e.g. Chiu and Said, 1994; Xia, 1997; Jia et al., 2016; Bahmanpouri et al., 2022). In Leopold (1953), the author refers to an unpublished work by the U.S. Geological Survey, where 2950 maximum point velocity measurements for different rivers are given. Their median value is 4.11 ft/s (1.25 m/s), the mean one 4.84 ft/s (1.47 m/s) and less than one percent of them exceed 13 ft/s. (3.96 m/s). Leopold also adds that "*the largest value of maximum point velocity in a natural river channel ever measured by stream-gaging personnel of the U. S. Geological Survey was about 22 ft/s*", i.e. 6.7 m/s. However, there are no guidelines as to what a reasonable range of basin scale velocity values is.

Based on Leopold (1953) and the hydraulic geometry studies led by Leopold and Maddock (1953) and considering flood flows, a consistent range of average velocity values can be supposed to be between 0.5 and 3 m/s. As shown in Fig. 3, all estimates fall within these boundaries, except for formulas with IDs = 11, 23 and 29 (see Fig. 3b) which, for larger basins, can provide velocities of up to 6 m/s.

The second and even more relevant aspect is to check which velocity behavior (i.e. increasing, decreasing or even constant) as a function of the *basin factor*, and thus the catchment size, would be the most appropriate. To this purpose, one can refer again to Leopold and Maddock (1953), where it is very clearly stated: "*Most geomorphologists are under the impression that the velocity of a stream is greater in the headwaters*

Table 3
m_1 and m_2 values controlling the relationship between basin area and *basin factor*, depending on the *L-S* pair used. Units are km^2 for *A* and km for $L/S^{0.5}$.

	$\frac{L_c}{\sqrt{S_c}}$	$\frac{L_c}{\sqrt{S_{LDP}}}$	$\frac{L_{LDP}}{\sqrt{S_{LDP}}}$	$\frac{L_{LDP}}{\sqrt{S_c}}$	$\frac{L_c}{\sqrt{S_{10-85,LDP}}}$	$\frac{L_{LDP}}{\sqrt{S_{10-85,LDP}}}$	$\frac{L_c}{\sqrt{S_{diff,LDP}}}$	$\frac{L_{LDP}}{\sqrt{S_{diff,LDP}}}$	$\frac{L_c}{\sqrt{S_b}}$	$\frac{L_{LDP}}{\sqrt{S_b}}$
m_1	0.751	0.488	0.266	0.446	1.44	0.968	1.07	0.67	0.63	0.334
m_2	1.18	1.3	1.39	1.26	1	1.05	1.11	1.18	1.43	1.54

Table 4
m_1 and m_2 values from other sites. Units are km^2 for A and km for $L/S^{0.5}$.

Formula ID	16	20	26	27
Location	UK	North Carolina (US)	California and Arizona (US)	Illinois (US)
Parameter	$\frac{L_c}{\sqrt{S_{10-85,LDP}}}$	$\frac{L_{LDP}}{\sqrt{S_{10-85,LDP}}}$	$\frac{L_c}{\sqrt{S_c}}$	$\frac{L_{LDP}}{\sqrt{S_{10-85,LDP}}}$
m_1	1.672	5.805	2.662	0.641
m_2	1.05	1.53	1.13	1.25

than in the lower reaches. The appearance of mountain stream, of course, gives the impression of greater kineticity than that observed in a large river downstream. The impression of greater velocity upstream stems in part from a consideration of river slopes which obviously are steeper in the upper than in the lower reaches. It will be recalled, however, that velocity depends on depth as well as on slope, as shown in the Manning equation. The fact that velocity increases downstream with mean annual discharge in the rivers studied indicates that the increase in depth overcompensates for the decreasing river slope. The magnitude of this rate of change becomes clear by comparison of the exponents of depth and slope. In the Manning equation the velocity depends on depth to the power 2/3 and on slope to the power 1/2". As Pilgrim (1977) also confirmed later, average velocities are found to increase slightly downstream, as the effects of the decrease in slope downstream are generally offset by adjustments in depth and hydraulic roughness. Similar results arise from Jowett (1998), where reach average velocities are calculated by hydraulic simulation on real cross-sections from 71 rivers in New Zealand. It is worth mentioning that the work of Shook et al. (2023) also reports increasing average velocities with watershed size, even though with a rather low correlation, probably due to the highly unusual landscape of the Canadian prairies.

Therefore, velocity can also be expressed as:

$$v = a_1 A^{a_2} \tag{16}$$

where a_2 values can be ≥ 0.

Fig. 4 shows the velocity values reported from Leopold and Maddock (1953) (green dots) and Jowett (1998) (purple dots) as a function of basin areas. The basin areas considered by Jowett, where available, have been retrieved from the work of Biggs et al. (1990). Fig. 4 also shows, for the two data sets, the fitted relationships as in Eq. (16). The exponent for growth rate of velocities with area is in both cases close to 0.1. As this result does not seem to depend on the specific study area, i.e. the works of Leopold and Maddock (1953) and Jowett (1998) cover completely different geographical areas, this value can be adopted as a reference. This allows us also to check whether the velocity estimates provided by each formula increases at a rate consistent with observations. To this end, a_1 and a_2 values for formulas providing increasing velocities are given in Table 5. a_2 values of 0.1 ± 0.025 are considered to be consistent with observations and are marked in bold in Table 5.

We note that there is a conceptual difference between the observed mean velocities at a cross-section, and the basin-scale velocity inferred using Eq (9). However, if one assumes downstream hydraulic geometry follows the power law behaviour of Leopold and Maddock (1953), then the two velocities are expected to be extremely similar. Therefore, we consider it is reasonable to use these observed velocities as a reference for basin-scale velocities.

It must be noted from Fig. 4 that the basins analyzed by Leopold and Maddock (1953) have a minimum size of about 30 km^2, as well as the New Zealand basins considered by Jowett (1998), which, except for a single watershed of about 4 km^2, are all bigger than 40 km^2. No indication is then given on the assumptions to be made for smaller basins. To the authors' knowledge, one of the few papers reporting measured velocities for small basins is that of Azizian (Azizian, 2019), where streamflow velocities are measured using tracers for basins of less than 20 km^2. However, this work is carried out on only 7 sub-basins.

Small basins are typically characterized by steep slopes, which would result in high average streamflow velocities. On the other hand, their hydrological response can be considered primarily governed by hillslope flow (Robinson et al., 1995; Hallema et al., 2016; Asano and Uchida, 2018), rather than channel flow, leading to a reduced average travel velocity. Velocities produced over small basins (i.e. over basin areas smaller than 30 km^2, for which no observations are available) must therefore be handled with greater caution. For the sake of safety, it is reasonable to assume that velocities might be nearly constant for small basins and, for this reason, formulas with IDs = 13, 25 and 28 (see Table 2) are not to be excluded a priori and their application can be relied on for low *basin factor* values.

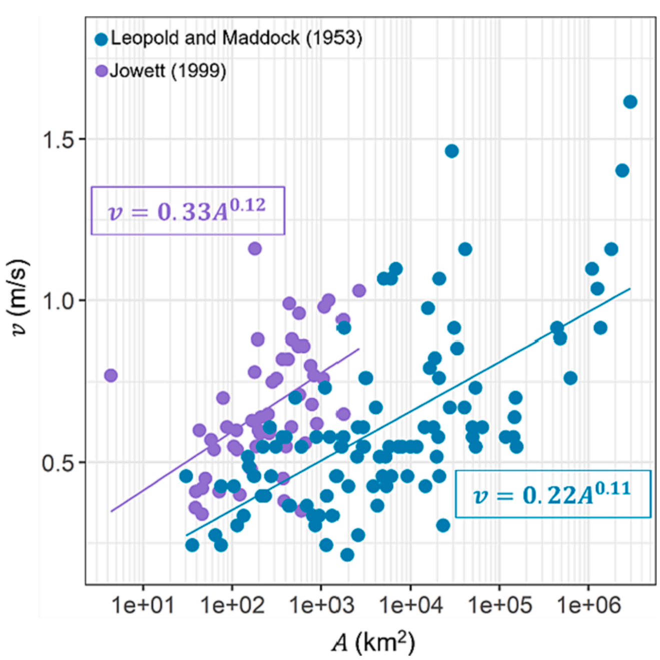

Fig. 4. "Observed" streamflow velocities in river basins in the US (Leopold and Maddock) and in New Zealand (Jowett).

Table 5
Parameters of the power law in Eq. (16) governing the relationship between velocity and basin area. Only formulas with a strictly increasing pattern in Fig. 3 are considered (i.e. formulas with IDs = 13, 25 and 28, which produce an almost constant velocity with the *basin factor*, are not included here). Units are km^2 for A and m/s for v. Cases where the exponent of the velocity growth rate is consistent with observations are highlighted in bold.

Formula ID	a_1	a_2
3	0.766	0.049
4	0.57	0.055
6	**0.962**	**0.095**
11	0.718	0.156
16	**0.496**	**0.124**
20	0.025	0.504
23	0.148	0.342
24	**0.49**	**0.086**
26	1.24	0.035
29	**1.08**	**0.116**

5. Theoretical bases to validate the inferred velocities and advices on calibrating new formulas

The aim of this section is to provide explanation to the different behavior of the formulas when applied to the sample of basins considered in this work. The increasing or decreasing velocities observed in Fig. 3 may reflect the existence of an underlying relationship between the scaling law linking length to slope and the formula used for the characteristic response time, i.e. the values of the α and β exponents in Eq. (1).

The expected relationship existing between length and slope is well established (Hack, 1957; Leopold, 1991). Typically, they are inversely proportional, and the following relationship can be written:

$$L = aS^b \tag{17}$$

where b is smaller than 0.

How length scales with slope for the basins investigated, i.e. the specific a and b values of Eq. (17), depend on which specific pair of length and slope is considered. Scatter plots for different combinations of length and slope parameters are shown in the section S.5 of the Supplementary Material (from Figures S5.1 to S5.10), while b values are given in Table 6. The value of b is affected by the choice of slope formulation, but not so much by the choice of length.

The length-slope scaling law expressed by Eq. (17) plays an important role also when considering whether the use of a formula in a certain geographical area is recommendable or not. In this regard, a separate discussion should be undertaken for the highly-cited Kirpich formula (ID = 10), which was calibrated on very different basins from the ones considered in this work (see Table A.2 of Appendix A). An in-depth discussion is provided in Appendix C, which suggests that the Kirpich's formula is unsuitable for application in the north-western Italy.

In the following sections, the relationships linking average velocities to the *basin factor* $L/S^{0.5}$ are derived analytically. The cases of *hydraulically consistent* or *non consistent* formulas are distinguished.

The implication of the results presented below is that a restricted range of $\alpha-\beta$ combinations for which v is an increasing function of area exists, depending on the value of the coefficient b, i.e. on the morphological properties of the watersheds investigated.

5.1. Hydraulically consistent formulas

In order to identify analytical relationships that justify the differing patterns in Fig. 3, our goal is to express v as a function of $\frac{L}{S^{1/2}}$. By dividing both terms of Eq. (17) by $S^{1/2}$, one obtains:

$$\frac{L}{S^{\frac{1}{2}}} = \frac{aS^b}{S^{\frac{1}{2}}} = aS^{b-\frac{1}{2}} \tag{18}$$

Eq. (18) can be then re-arranged to make L the subject. By raising Eq. (18) to the power of $\frac{b}{b-\frac{1}{2}}$, and dividing and multiplying the right-hand term by a, one finds:

$$\left(\frac{L}{S^{\frac{1}{2}}}\right)^{\frac{b}{b-\frac{1}{2}}} = a^{\frac{b}{b-\frac{1}{2}}-1} aS^b \tag{19}$$

By assuming $d = a^{\frac{b}{b-\frac{1}{2}}-1}$ and recalling Eq. (17), one obtains:

$$L = \frac{1}{d}\left(\frac{L}{S^{\frac{1}{2}}}\right)^{\frac{b}{b-1/2}} \tag{20}$$

Table 6
Empirical b values for different length-slope pairs. Unit for L is km, while S is dimensionless.

	S_c	S_{LDP}	$S_{10-85,LDP}$	$S_{diff,LDP}$	S_b
L_c	−1.2	−1.82	−0.78	−1.06	−1.45
L_{LDP}	−1.1	−1.67	−0.72	−0.97	−1.34

According to Eq. (1), Eq. (9) can also be written as:

$$v = \frac{L^{1-\alpha}S^{\beta}}{1.5\gamma} \tag{21}$$

By substituting Eq. (20) into Eq. (21), it follows:

$$v = \frac{\frac{1}{d}\left(\frac{L}{S^{\frac{1}{2}}}\right)^{\frac{b}{b-1/2}}}{\gamma\left(\frac{L}{S^{\frac{1}{2}}}\right)^{\alpha}} \tag{22}$$

and thus:

$$v = \gamma\left(\frac{L}{S^{\frac{1}{2}}}\right)^{\frac{b}{b-1/2}-\alpha} \tag{23}$$

According to Eq. (23), the sign of v as a function of the *basin factor* depends on the following condition:

$$\begin{cases} \alpha < \dfrac{b}{b-1/2} & (+) \\ \alpha > \dfrac{b}{b-1/2} & (-) \end{cases} \tag{24}$$

In order to apply the criterion given in Eq. (24), the formula should have length and slope exponents in a 2:1 ratio, and not contain other parameters correlated to length and slope. Based on b values in Table 6, threshold values $\frac{b}{b-1/2}$ are given in Table 7 and compared with the α value of each *hydraulically consistent* formula considered. Eq. (24) is verified in all the cases.

One of the interesting implications is that the Chezy and Gauckler-Strickler-Manning equations, both having $\alpha = 1$, which is always greater than the threshold values found, cannot produce formulas where velocity increases with basin size. Bearing in mind that velocity is estimated here as the ratio between a length and a time, the condition $\alpha = 1$ means that the length parameter is missing in the definition of v(see Eq. (21)). We then have that the velocity is a function of the slope only, which is almost always decreasing with the basin area. This also confirms that Chezy's ideal channel with constant slope, width and roughness does not exist in reality and the assumption of constant discharge underlying Chezy's law contradicts what was observed in Leopold and Maddock (1953).

This aspect may sound inconsistent to the reader. However, it is worth specifying that the 2:1 ratio between the length and slope exponents is still a useful tool for classification and screening procedures such as those carried out in this work. Formulas in which the α and β exponents are in a 2:1 ratio are easier to understand, both because the quantity $L/S^{0.5}$ is correlated to the basin area, and because the analytical relationship that explains the behavior of these formulas in relation to basin morphology (Eq. (24)) is formally simple.

5.2. Non hydraulically consistent formulas

A more general analysis is possible for formulas which do not have 2:1 exponents for length and slope. Following the same procedure as in the previous case (all details are given in section S.6 of the Supplementary Material), one obtains:

$$\begin{cases} \alpha < 1 + \dfrac{\beta}{b} & (+) \\ \alpha > 1 + \dfrac{\beta}{b} & (-) \end{cases} \tag{25}$$

In the special case where $\beta = \frac{\alpha}{2}$, Eq. (25) simplifies to Eq. (24). As in the previous case, this reasoning can be applied only to formulas that do not contain, in addition to L and S, other correlated parameters.

The only two *non-hydraulically consistent* formulas which contain no additional input variables are the Temez (ID = 26) and USGS (ID = 27) formulas, for which results are given in Table 8. The criterion provided in Eq. (25) is fulfilled in both cases.

Table 7
Verification of Eq. (24) for *hydraulically consistent* formulas that do not include any variables related to length and slope. From left to right: formula identification, input parameters, value of the length exponent α, threshold value $\frac{b}{b-1/2}$, slope of the regression line between predicted velocity and the *basin factor*, verification outcome.

Formula ID	Length parameter	Slope parameter	α value	Threshold value for α	Slope of the v-$L/S^{0.5}$ line	Compliance with Eq. (24)
6	L_{LDP}	S_c	0.64	0.69	+	Yes
10	L_{LDP}	$S_{diff,LDP}$	0.8	0.66	–	Yes
13	L_{LDP}	$S_{10-85,LDP}$	0.64	0.59	–	Yes
14	L_{LDP}	$S_{10-85,LDP}$	0.74	0.59	–	Yes
16	L_c	$S_{10-85,LDP}$	0.47	0.61	+	Yes
19	L_{LDP}	S_c	1	0.69	–	Yes
20	L_{LDP}	$S_{10-85,LDP}$	0.5	0.59	+	Yes
28	L_{LDP}	S_c	0.79	0.69	–	Yes

For the above-mentioned reasons, Eq. (25) cannot be applied for the Bocchiola et al. (ID = 3), Bransby Williams (ID = 4), Linsley (ID = 11), Schulz (ID = 23), SCS (ID = 24) and Sheridan (ID = 25) formulas. The Curve Number, which is used as an input in the Bocchiola et al. and SCS formulas, shows a correlation with the basin slope (see Figure S7.1 in section S.7 of the Supplementary Material). The main channel length L_c and the longest drainage path length L_{LDP} are strongly correlated with the basin area (see Figures S2.12 and S2.13 of section S.2 of the Supplementary Material), which is one of the inputs of the Bransby-Williams formula, as well as with the L_{ca} (see Figures S7.2 and S7.3 of the section S.7 the Supplementary Material), included in both Schulz and Sheridan formula.

As an example, Table 8 shows the results of the condition expressed by Eq. (25) if applied to the Bocchiola et al. (ID = 3) and SCS (ID = 24) formulas. It can be seen that the Bocchiola et al. formula is consistent with the derivations described above as it gives increasing velocities and the α exponent is lower than the threshold value, even if only slightly. The SCS formula, on the other hand, does not comply with Eq. (25). This may be due to the fact that although the structure of the two formulas is the same, the factor $(1 + CN_s)$ is raised to the power 0.13 in the first case, and 0.8 in the second. In the SCS formula the effect of the correlation between CN and S_b is therefore much more influential.

6. Guidance towards the selection of robust formulas and conclusions

The main objective of this study was to use hydraulic reasoning to classify the large number of formulas available for estimating flood response time in ungauged basins, both to provide concrete guidance to practitioners and to suggest a methodology to identify robust formulas that is based on observed velocities, rather than observed times. Although other review-type works are available on this topic, an approach based on velocities has not been explored before in the literature. To this end 29 formulas, all containing a basin length and slope parameter, have been selected and the behavior of some of them when applied to 135 basins in north-western Italy has been investigated.

To allow the application, an in-depth assessment of the input variables to be used in each formula was first undertaken. A careful description of each formula is then provided, including inputs and units.

Some indicators that help to quantify the robustness of formulas have been then defined. The initial step in the classification process involves the hydraulic-based consistency of the formula structure, according to hydraulic relations, such as Chezy's, Manning's or the kinematic wave model. Although this is not a strict requirement, hydraulic formulas imply that length and slope exponents are in a 2:1 ratio. 13 formulas, from the 29 collected, have been recognized to be *hydraulically consistent*.

After this preliminary overview, criteria were examined that would allow the identification of robust formulas, considering the significance of values of the average spatio-temporal velocities that each formula produces, calculated from the associated flood wave celerities. The following research questions have been then addressed:

i) Are the values of velocity sufficiently consistent with observations?
ii) Does predicted velocity increase with basin area, as observations show?
iii) Is the rate at which velocity increases with basin area consistent with observations?

An overview on how formulas perform when applied to the watersheds investigated, according to the above criteria, is given in Table 9. Only two formulas satisfy all the aspects investigated in terms of flow velocities, while also being *hydraulically consistent*, i.e. Chow's formula (ID = 6) and NERC's formula (ID = 16). However, formulas that do not seem to comply with the 2:1 α/β ratio supported by the hydraulic reasoning are not to be excluded a priori. Indeed, it may happen that some formulas provide velocities that are fully consistent with observations, in terms of either their magnitude and their behavior with the *basin factor* (and thus with the watershed size) even though the $\frac{\alpha}{\beta}$ ratio is not fulfilled. In such cases, the non-compliance with the 2:1 ratio may only be apparent, as additional variables, which are correlated with length and slope, must be accounted for. An example is the SCS formula (ID = 24), which can also be considered a good formulation according to the criteria adopted in this work.

Also the *hydraulically consistent* formulas of McEnroe and Zhao (1999) (ID = 13) and Watt and Chow (ID = 28) are not to be rejected, despite they do not provide velocities strictly increasing with basin area, but almost constant. As explained in section 5.1, a constant velocity as the area increases can be assumed for safety reasons for small basins.

It is worth specifying that the above formulas are recommended here

Table 8
Verification of Eq. (25) for *non hydraulically consistent* formulas. Formulas with IDs = 26 and 27 do not include any variables related to length and slope, formulas with IDs = 3 and 24 include the *CN* variable. From left to right: formula identification, input parameters, value of the length exponent α, value of the slope exponent β, threshold value $1 + \frac{\beta}{b}$, slope of the regression line between predicted velocity and the *basin factor*, verification outcome.

Formula ID	Length parameter	Slope parameter	α value	β value	Threshold value for α	Slope of the v-$L/S^{0.5}$ line	Compliance with Eq. (25)
26	L_c	S_c	0.76	0.19	0.84	+	Yes
27	L_{LDP}	$S_{10-85,LDP}$	0.875	0.181	0.75	–	Yes
3	L_c	S_b	0.82	0.2	0.86	+	Yes
24	L_{LDP}	S_b	0.8	0.5	0.63	+	No

Table 9
Overall behavior of the tested formulas. Chow (ID = 6) and NERC (ID = 16) formulas satisfy all criteria. SCS formula (ID = 24) provides robust results in terms of velocity estimates even though the 2:1 ratio is apparently not complied with. McEnroe and Zhao (1999) (ID = 13) and Watt and Chow (ID = 28) formulas provide reliable results, particularly for small basins.

Formula ID	Consistent α/β ratio	1st criterion: Consistent velocity values	2nd criterion: Consistent velocity behavior	3rd criterion: Consistent velocity growth rate
3	No	Yes	Yes	No
4	No	Yes	Yes	No
6	**Yes**	**Yes**	**Yes**	**Yes**
10	Yes	Yes	No	–
11	No	No	Yes	No
13	**Yes**	**Yes**	**Yes**	**(constant)**
14	Yes	Yes	No	–
16	**Yes**	**Yes**	**Yes**	**Yes**
19	Yes	Yes	No	–
20	Yes	Yes	Yes	No
23	No	No	Yes	No
24	**No**	**Yes**	**Yes**	**Yes**
25	No	Yes	Yes	(constant)
26	No	Yes	Yes	No
27	No	Yes	No	–
28	**Yes**	**Yes**	**Yes**	**(constant)**
29	No	No	Yes	Yes

based on our considerations about predicted velocities. On the other hand, it is up to the reader to judge whether the number of watersheds used in calibrating the formula, or its range of application is reasonable or suitable for one's purpose.

With the aim of motivating the different behavior of the predicted velocity estimates, the relationships between velocities and the basin morphological scaling laws have been theoretically explored. This allowed us to identify the range of length and slope exponents in the characteristic time formulas for which velocity increases with basin areas, both for formulas in compliance with or not to the hydraulic consistency. This type of analysis allowed us to highlight that formulas should not be applied to basins whose length-slope scaling is different to the basins used for calibration. An example is that of the Kirpich formula (ID = 10), which seems to be unsuitable for the study area considered here.

The analytical relationships derived in this work may not only be adopted for classifying existing formulas but can also provide guidelines to undertake the calibration of new formulas, making them provide reasonable velocity estimates. In that case, one may expect that the specific combinations of length and slope exponents can differ from those found in this work, depending on the specific region of calibration of the formula, but the derivations and equations shown here will have a general validity.

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CRediT authorship contribution statement

Giulia Evangelista: Conceptualization, Methodology, Validation, Formal analysis, Investigation, Data curation, Writing – original draft, Writing – review & editing, Visualization. **Ross Woods:** Conceptualization, Methodology, Formal analysis, Investigation, Writing – review & editing, Supervision. **Pierluigi Claps:** Conceptualization, Methodology, Formal analysis, Investigation, Resources, Writing – review & editing, Supervision.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A. Characterization of formulas

Table A1
Summary of equations used to estimate different time parameters. For symbols, the reader can refer to the Nomenclature.

ID	Reference	Equation	Units
1	Aron et al. (1991)	$t_c = 0.93\frac{R_s{}^{5/12}n^{3/4}L_{LDP}{}^{7/12}}{\delta^{1/2}i^{1/4}S_c{}^{3/8}}$	t_c in min $LLDP$ in m i in mm/h R_s, δ, S_c, n= dimensionless
2	Askew (1970)	$t_L = 0.877L_{LDP}^{0.8}S_b^{-0.33}q_{wm}^{-0.23}$	t_L in hours L_{LDP} in km S_b dimensionless q_{wm} in m^3/s
3	Bocchiola et al. (2003)	$t_L = 0.26L_c^{0.82}S_b^{-0.2}(1+CN_S)^{0.13}$	t_L in hours L_c in km S_b in % CN_S in mm
4	Bransby Williams (1922)	$t_c = 14.467L_{LDP}S_{LDP}^{-0.2}A^{-0.1}$	t_c in minutes L_{LDP} in km A in km^2 S_{LDP} dimensionless
5	Carter (1961)	$t_L = 3.1L_{LDP}^{0.6}S_{c,w-L}^{-0.3}$	t_L in hours L_{LDP} in mi $S_{c,w-L}$ in ft/mi

(continued on next page)

Table A1 (*continued*)

ID	Reference	Equation	Units
6	Chow (1962)	$t_p = 0.00236 L_{LDP}^{0.64} S_c^{-0.32}$	t_p in hours L_{LDP} in ft S_c in %
7	Espey et al. (1966)	$t_{peak} = 2.65 L_{LDP}^{0.12} S_{c,w-L}^{-0.52}$	t_{peak} in minutes L_{LDP} in ft $S_{c,w-L}$ dimensionless
8	Espey et al. (1966)	$t_{peak} = 20.8 E_f L_{LDP}^{0.29} S_{c,w-L}^{-0.11} A_{imp}^{-0.6}$	t_{peak} in minutes E_f dimensionless L_{LDP} in ft $S_{c,w-LDP}$ dimensionless A_{imp} in %
9	Kennedy and Watt (1967)	$t_L = 8.8 L_{LDP}^{0.75} S_c^{-0.375} W^{1.3}$	t_L in hours L_{LDP} in mi S_c in ‰ W dimensionless
10	Kirpich (1940)	$t_c = 0.0058 L_{LDP}^{0.8} S_{diff,LDP}^{-0.4}$	t_c in minutes L_{LDP} in ft $S_{diff,LDP}$ dimensionless
11	Linsley et al. (1958)	$t_p = a(L_{LDP} L_{ca})^{0.38} S_c^{-0.19}$	t_p in hours L_{LDP} in mi L_{ca} in mi S_c in ft/mi
12	McCuen et al. (1984)	$t_c = 0.01462 L_{LDP}^{0.5552} S_c^{-0.2070} i_2^{-0.7164}$	t_c in hours L_{LDP} in ft S_c in ft/mi i_2 in in/hours
13	McEnroe and Zhao (1999)	$t_p = 0.086 L_{LDP}^{0.64} S_{10-85,LDP}^{-0.32}$	t_p in hours L_{LDP} in km $S_{10-85,LDP}$ dimensionless
14	McEnroe and Zhao (2001)	$t_p = 0.058 L_{LDP}^{0.74} S_{10-85,LDP}^{-0.37} e^{-3.5A_{imp}}$	t_p in hours L_{LDP} in km $S_{10-85,LDP}$ dimensionless A_{imp} dimensionless
15	McEnroe and Zhao (2001)	$t_p = 0.106 L_{LDP}^{0.63} S_{10-85,LDP}^{-0.315} e^{-0.1RD}$	t_p in hours L_{LDP} in km $S_{10-85,LDP}$ dimensionless RD in km^{-1}
16	NERC (1975)	$t_{peak} = 2.8 L_c^{0.47} S_{10-85,LDP}^{-0.235}$	t_{peak} in hours L_c in km $S_{10-85,LDP}$ in m/km
17	Overton and Meadows (1976)	$t_c = 0.928 i^{-0.4} n^{0.6} L_{LDP}^{0.6} S_{LDP}^{-0.3}$	t_c in minutes i in in/h L_{LDP} in ft S_{LDP} dimensionless
18	Papadakis and Kazan (1987)	$t_c = 0.66 L_{LDP}^{0.5} n^{0.52} S_{LDP}^{-0.31} i^{-0.38}$	t_c in minutes L_{LDP} in ft S_{LDP} dimensionless i in in/h
19	Pezzoli (1970)	$t_c = 0.055 L_{LDP} S_c^{-0.5}$	t_c in hours L_{LDP} in km S_c dimensionless
20	Putnam (1972)	$t_L = 0.49 L_{LDP}^{0.5} S_{10-85,LDP}^{-0.25} A_{imp}^{-0.57}$	t_L in hours L_{LDP} in mi $S_{10-85,LDP}$ in ft/mi A_{imp} dimensionless
21	Rossi (1974)	$t_L = 0.77 L_c^{0.295} S_{c,w-T\&S}^{-0.147}$	t_L in hours L_c in km $S_{cw-T\&S}$ dimensionless
22	Schaake et al. (1967)	$t_L = 1.05 L_c^{0.24} S_c^{-0.16} A_{imp}^{-0.26}$	t_L in minutes L_c in ft S_c in % A_{imp} dimensionless
23	Schulz (1969)	$t_{peak} = 1.9(L_{LDP} L_{ca})^{0.162} S_{diff,LDP}^{-0.081}$	t_{peak} in hours L_{LDP} in km L_{ca} in km $S_{diff,LDP}$ dimensionless
24	SCS (1954)	$t_p = \frac{L_{LDP}^{0.8}(1+CN_S)^{0.7}}{1900 S_b^{0.5}}$	t_p in hours L_{LDP} in ft S_b in % CN_S in in
25	Sheridan (1994)	$t_{peak} = 0.63(L_c L_{ca})^{0.4} S_c^{-0.2}$	t_{peak} in hours L_c in km L_{ca} in km S_c dimensionless

(*continued on next page*)

Table A1 (*continued*)

ID	Reference	Equation	Units
26	Temez (1987)	$t_L = 0.126L_c^{0.76}S_c^{-0.19}$	t_L in hours L_c in km S_c dimensionless
27	USGS (2000)	$t_c = 1.54L_{LDP}^{0.875}S_{10-85,LDP}^{-0.181}$	t_c in hours L_{LDP} in mi $S_{10-85,LDP}$ in ft/mi
28	Watt and Chow (1985)	$t_p = 0.000326L_{LDP}^{0.79}S_c^{-0.395}$	t_p in hours L_{LDP} in m S_c dimensionless
29	Williams (1968)	$t_{peak} = 0.144L_{LDP}^{0.935}S_{LDP}^{-0.369}R_e^{1.486}$	t_{peak} in hours L_{LDP} in mi S_{LDP}, R_e dimensionless

Table A2
Details about the calibration of each equation. For symbols, the reader can refer to the Nomenclature.

ID	N° of basins used for calibration	Calibration range	Notes	Google Scholar Citations (as to 2.03.2023)
1	Analytical approach	Not calibrated	• Based on the kinematic wave equation. • n is the Manning coefficient.	22
2	5	$A = 0.4$–90 km^2 $L_{LDP} = 0.7$–25 km $S_b = 0.05$–0.22	• Calibrated from about 200 events in small basins near Sydney, Australia. • The average basin slope is computed using the grid-contour intersection method: $S_b = \frac{N \bullet h}{l}$ where l is the total length of grid lines within the catchment, h is the contour interval and N is the total number of intersections	78
3	43	$A = 42$–4800 km^2 $L_c = 9$–142 km $S_b = 0.05$–0.26	• Calibrated from 58 events in basins all over Italy. • S_b is computed from a DEM with a N-E resolution of 220x230m. • The fitting performance is $R^2 = 0.89$.	8
4	No information	No information	• No information about how this formula was derived, nor on the data used. • The author stated “this formula gives a somewhat more rapid concentration than actually takes place in most instances, but such inaccuracy as there is on the right side”. • The slope of the drainage path is defined as the “average number of feet fall per 100 ft along the greatest length of the watershed”.	65
5	20	$A = 10$–1415 km^2	• Data of urban basins in the US. • The average slope refers to the “weighted slope of an order of 3 or greater of all stream channels in the basin”.	235
6	20	$A = 0.01$–18.5 km^2 $L_{LDP} = 0.2$–8 km $S_c = 0.005$–0.06	• Calibrated from 53 events in basins in Illinois, Ohio, Missouri, Wisconsin, Indiana, Iowa, and Nebraska. • The slope does not refer to the weighted average slope but to the slope of the “straight line of best fit” in the plane elevation-distance. The author affirms that the results are almost the same.	183
7	11	$L_{LDP} = 1$–8 km $S_{c,w-L} = 0.008$–0.15	• Data of rural watersheds in Texas, New Mexico and Oklahoma. • The fitting performance is $\rho = 0.972$	150
8	22	$L_{LDP} = 0.05$–17 km $S_{c,w-L} = 0.006$–0.1 $A_{imp} = 2.7$–100 %	• Data of urban watersheds in Texas, New Mexico and Oklahoma. • $E_f = \begin{cases} 0.6 \text{ for extensive channel improvement} \\ 0.8 \text{ for cleaning and enlargement of existing channel} \\ 1 \text{ for natural channel conditions} \end{cases}$ • The fitting performance is $\rho = 0.954$.	150
9	12	$A = 60$–320 km^2 $L_{LDP} = 8$–30 km $S_c = 0.001$–0.007	Data of watersheds in southern Ontario.	20
10	6	$A < 0.45$ km^2 $L_{LDP} = 0.1$–1.2 km $S_{diff,LDP} = 0.027$–0.098	• Calibrated from 10 rainfall events in 1918 for small rural watersheds in Madison County (Tennessee), having well-defined drainage channels and a quite hilly topography. Silt loamy soil. Data from Ramser (1927). • t_c is computed as “the time required for the water in the channel at the gaging station to rise from the low to the maximum stage”.	844
11	18	$A = 6.5$–1670 km^2	• Data from watersheds in California. • 3 equations are provided: a=0.72 for foothill drainage areas, a=1.2 for mountain drainage areas, a=0.35 for valley drainage areas.	3398
12	39	$A = 0.4$–16 km^2 $L_{LDP} = 1$–9 km $S_c = 0.0007$–0.031	• Data from urban watersheds in the US. • The “observed” t_c is computed using the SCS velocity method.	219
13	19	$A = 2.15$–27 km^2 $L_{LDP} = 2.3$–15 km$S_{10-85,LDP} = 0.001$–0.02	• Calibrated from 200 events for small rural watersheds in Kansas. • The fitting performance is $R^2 = 0.91$.	19
14	14	$A = 0.7$–73 km^2 $L_{LDP} = 1.5$–24 km$S_{10-85,LDP} = 0.0025$–0.016 $A_{imp} = 1$–40 %	• The watershed should not contain any lakes or detention sites. • The urban development within the watershed should be reasonably well distributed. • The impervious areas were computed from the coverages of building outlines, edges lines for roads and parking lots, and the coverages of driveway centerlines. • The fitting performance is $R^2 = 0.976$.	11
15	14	$A = 0.7$–73 km^2 $L_{LDP} = 1.5$–24 km$S_{10-85,LDP} = 0.0025$–0.016 $RD = 0.8$–16.5 km/km^2	• The watershed should not contain any lakes or detention sites. • The urban development within the watershed should be reasonably well distributed. • The total length of roads were computed from the CENT coverages of road centerlines. • The fitting performance is $R^2 = 0.982$.	11

(*continued on next page*)

Table A2 (*continued*)

ID	N° of basins used for calibration	Calibration range	Notes	Google Scholar Citations (as to 2.03.2023)
16	132	$A < 500$ km^2 (not strictly followed)	• Calibrated from nearly 1500 events for basins in UK (10 events, on average, are available for each basin). • The selected basins display some evidence of a short term response to heavy rainfall.	218
17	Analytical approach	Not calibrated	Based on Manning's kinematic solution.	372
18	87	$A < 2$ km^2 (for natural watersheds)	• Data of small rural basins in 22 states in the US, provided by the USDA Agricultural Research Service, and experimental data from the US Army Corps of Engineers (rainfall tests on airfield strips at Santa Monica Municipal Airport), the Colorado State University and the University of Illinois (rainfall tests on experimental basins). • The "observed" t_c is computed as the time difference between the end of rainfall and the inflection point of the hydrograph. • 375 data points are used to fit the equation. • The fitting performance is $R^2 = 0.941$.	53
19	Analytical approach	Not calibrated	• Coming from the Chezy's formula. • Tested on basins in the Valle d'Aosta region (Italy).	1
20	118	$A < 400$ km^2 $L_{LDP}/\sqrt{S_{10,85,LDP}} = 0.1$–$9$ $A_{imp} = 1$ % (for undeveloped basins) to 100 % (for urban basins)	• Data for 44 of these basins were collected in the Piedmont region of North Carolina. Data for the remainder of the stations were obtained from a report by Anderson (1970). • 6 to 20 storm events are selected for each basin. Selection criteria: 1. uniformly distributed rainfall intensities over the basin area 2. rainfall duration shorter than the expected lag time 3. single-peak flood hydrograph. • The impervious cover is determined by visual inspection in the drainage basin.	7
21	7	$A = 40$–1650 km^2 $L_c = 13$–105 km $S_{c,w-T\&S} = 0.006$–0.03	• Data from watersheds in Basilicata (southern Italy). • 4 to 12 rainfall events are available for each basin.	9
22	19	$A < 1$ km^2 $L_c < 2$ km $S_c = 0.008$–0.06 $A_{imp} = 8.7$–100 %	• Data from watersheds in Baltimore (Maryland) and Newark (Delaware), USA. • The slope is found by dividing the difference in elevation between the upstream and downstream ends of the main channel by the length of the channel in hundreds of feet. • The fitting performance is $\rho = 0.85$.	130
23	5	$A = 24$–1060 km^2 $L_{LDP} = 10$–80 km $S_{diff,LDP} = 0.01$–0.1	• Data from watersheds in Thailand.	No citations
24	No information	$A < 8$ km^2 CN = 50–95	No information about how this formula was derived.	88
25	9	$A = 2.6$–335 km^2 $L_c = 2.4$–42 km $S_c = 0.001$–0.004	• Calibrated from 75 events in low gradient basins in Georgia and Florida (USA). • The fitting performance is $R^2 = 0.956$.	49
26	25	$A = 6$–11162 km^2$L_c =$ 4.7–240 km $S_c = 0.006$–0.2	• Modified version of the US Corps of Engineers formula. • Data from watersheds in California and Arizona (US), used by the US Corps of Engineers.	308
27	39	$A < 4$ km^2 $S_{10-85,LDP} = 0.002$–0.045 $L_{LDP} = 0.3$–6 km	• Calibrated from 121 Events in small rural watersheds in Illinois. • The "observed" t_c is computed as the time difference between the end of rainfall and the inflection point of the hydrograph. • The fitting performance is $R^2 = 0.73$.	97
28	44	$A = 0.005$–5840 km^2$L_{LDP} =$ 0.1–200 km $S_c = 0.001$–0.09	• Data from watersheds in North America. • Not tested for very flat basins, urban basins or those having large lakes. • The fitting performance is $R^2 = 0.96$.	74
29	9	$A = 0.7$–45.5 km^2$L_{LDP} =$ 1.5–13.5 km $S_{LDP} = 0.034$–0.017	• Calibrated from 73 storm events from 1940 to 1965. • Average annual precipitation is 840 mm and average annual runoff is 142 mm. • The fitting performance is $\rho = 0.99$.	12

Appendix B. Influence of varying the input parameters on time estimates

Since the length variables L_c and L_{LDP} are correlated by a strong linear relationship (see Figure S2.1 in the Supplementary Material), many of the errors in response time estimation may be due to errors in the choice of slope parameter. In Fig. B.1 the average slope of the longest drainage path S_{LDP}, is compared with simplified formulations, whether $S_{diff,LDP}$ and $S_{10-85,LDP}$ computed according to Eqs. (S1-2) and (S1-3), respectively (see section S.1 of the Supplementary Material). It can be observed that simplified formulations tend to provide higher estimates for high slope values as they may place great emphasis on the steep slopes in the headwaters region, which are hydraulically quite far from the outlet. Even larger differences occur if the average stream path slope, however calculated, is substituted for the average basin slope (see e.g. Figure S2.8 in the Supplementary Material).

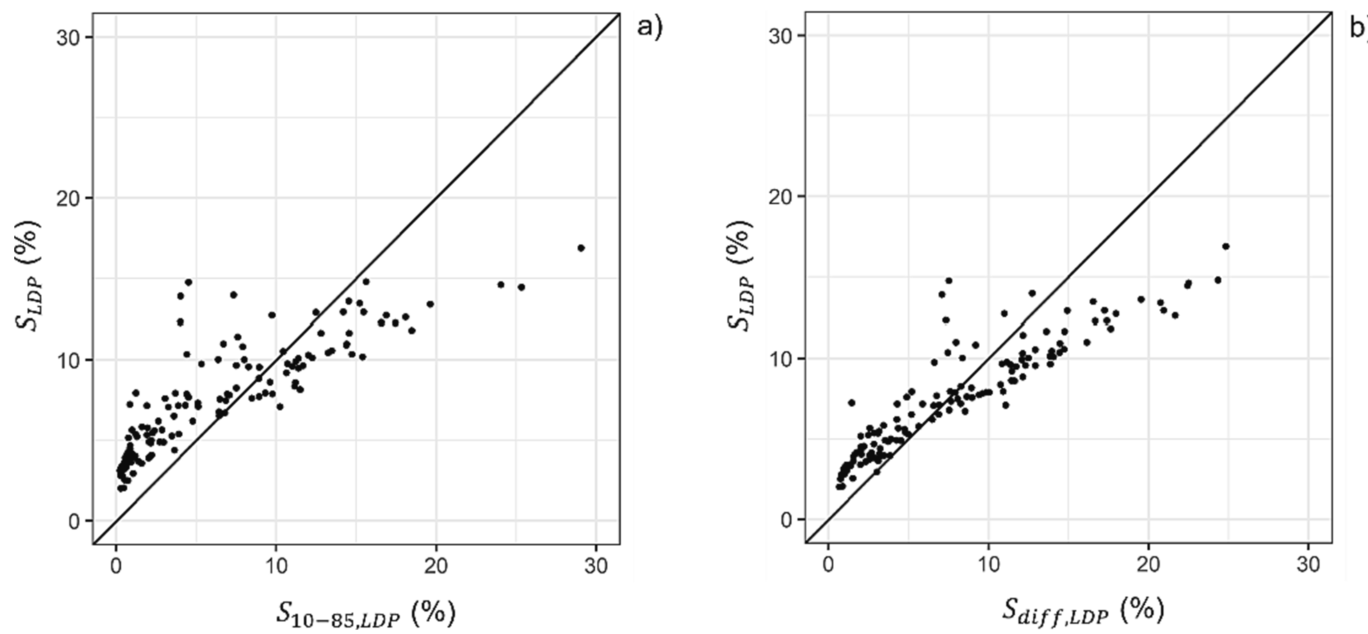

Fig. B1. Comparison of slope values of the longest drainage path computed according to different definitions.

The sensitivity of the response time to the adopted slope parameter was then investigated. As an example, this analysis is focused on those formulas containing the slope of the longest drainage path and for which data are available (IDs 4, 10, 13, 14, 16, 19, 20, 23, 27, 29). The formulations we refer to in order to quantify sensitivity are therefore S_{LDP}, $S_{10-85,LDP}$ and $S_{diff,LDP}$. The formula's original slope parameter has been replaced with alterative slope formulations, resulting in a new estimate of t, called $t_{modified}$. Then, the variation Δt of $t_{modified}$ as compared to the response time provided by the formula in its original equation, $t_{original}$, was calculated, according to:

$$\Delta t = \frac{t_{modified} - t_{original}}{t_{original}} \tag{B.1}$$

In Table B.1, the maximum positive and negative Δt values are provided for each formula. It can be observed that when moving from the S_{LDP} parameter to the simplified ones, one obtains a significant overestimation of t for small slope values and a slight underestimation as the slope increases. On the other hand, an underestimation of the response time is generally observed for small slope values when substituting $S_{10-85,LDP}$ or $S_{diff,LDP}$ with S_{LDP}. In all cases, with the exception of the Schulz formula (ID = 23), whose results are quite stable, the estimated response time for a given basin can vary by more than 0.25. In the case of Pezzoli's formula (ID = 19), Δt can reach positive values of up to a factor of 2.

In the Supplementary Material (section S3), from Figure S3.1 to Figure S3.20, Δt values are depicted as a function of S_{LDP} for each of the formula considered.

Table B1
Maximum positive and negative Δt values obtained when modifying the slope parameter formulation. Formulas containing the slope of the longest drainage path are tested. Cases where Δt values exceed ± 0.25 are highlighted in red, while values greater than ± 1 are shown in bold.

Formula ID	Original slope parameter	Alternative slope parameter		
		S_{LDP}	S_{10-85}	$S_{diff,LDP}$
4	S_{LDP}		+0.66 −0.11	+0.38 −0.1
10	$S_{diff,LDP}$	+0.24 −0.47	**+1.07** −0.1	
13	$S_{10-85,LDP}$	+0.2 −0.56		+0.08 −0.44
14	$S_{10-85,LDP}$	+0.23 −0.61		+0.1 −0.49
16	$S_{10-85,LDP}$	+0.14 −0.45		+0.06 −0.35
19	S_{LDP}		**+2.56** −0.25	**+1.23** −0.24
20	$S_{10-85,LDP}$	+0.15 −0.47		+0.06 −0.36
23	$S_{diff,LDP}$	+0.04 −0.12	+0.16 −0.02	
27	$S_{10-85,LDP}$	+0.11 −0.37		+0.05 −0.28
29	S_{LDP}		**+1.55** −0.19	+0.81 −0.18

Appendix C. A singular case: The Kirpich formula

Fig. C.1 shows some scatter plots describing how the Kirpich formula works. Even though the response times used to calibrate the formula are consistent with those that the formula yields for the basin sample in the study area (Fig. C.1a), when moving from time estimates to velocity estimates one can note something interesting (Fig. C.1b) as, for the same basin factor, the velocities resulting from the observed response time on which the formula is calibrated are about twice as fast as those obtained using the formula on the sample of basins analyzed. The reason for such behavior is to be found among the morphological features of the basins used to calibrate the formula. In Fig. C.1c the relationship between L_{LDP} and $S_{diff,LDP}$, which are the inputs of the Kirpich formula, is given for basins in north-western Italy (grey dots). Among them, basins having an unusual behavior (red dots) can be recognized. The unusual feature lies in the small values of both the slope $S_{diff,LDP}$ and the length L_{LDP}(remember that length and slope are typically inversely proportional, according to Eq. (17)). Looking at Fig. C.1c, the watersheds used to calibrate the Kirpich formula (green dots) can also be included among those having an anomalous behavior and thus yielding much lower velocities compared to the ones the formula gives for the basin sample in north-western Italy. Despite its wide popularity, the Kirpich formula should be handled with caution.

This example allows us to remark that formulas should not be applied to basins whose length-slope scaling is quite different from the basins used for calibration.

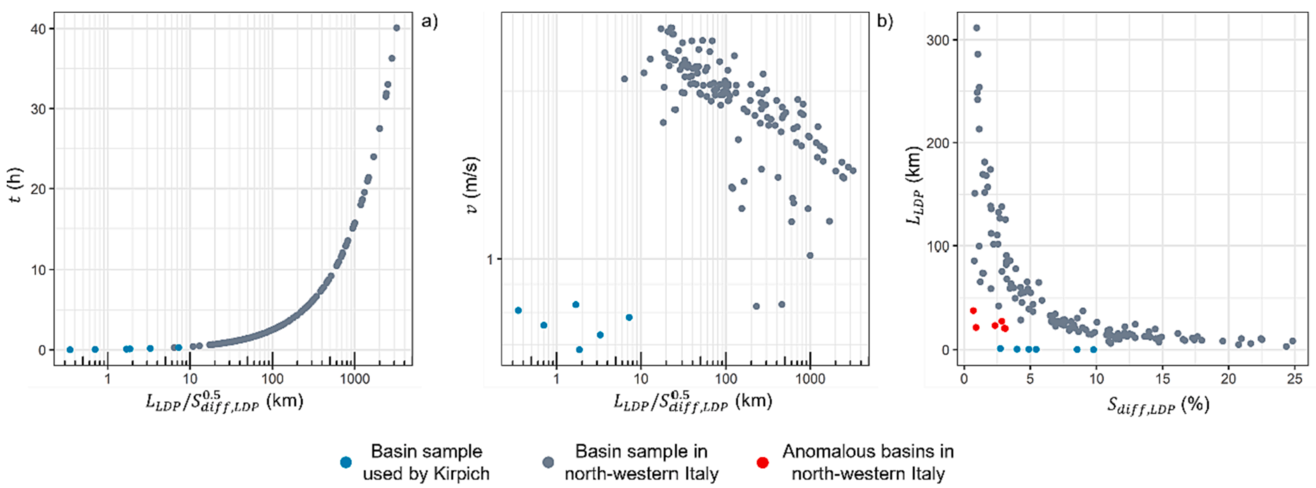

Fig. C1. Kirpich formula behavior. a) Times used to calibrate the formula, compared with those returned by the formula when applied to the basins in north-western Italy. b) Velocities obtained from the times used to calibrate the formula, compared with those obtained using the formula. c) Morphological characteristics of basins in north-western Italy, compared with those of the basins used for calibration. The red dots represent basins in the study area with anomalous behaviour, i.e. Banna at Santena, Borbore at San Damiano d'Asti, Bormida di Mallare at Ferrania, Bormida di Millesimo at Murialdo, Bormida di Spigno at Valla and Niguglia at Omegna.

Appendix D. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jhydrol.2023.130409.

References

Anderson, D. G., 1970. Effects of urban development on floods in Northern Virginia: U.S. Geol. Survey Water-Supply Paper 2001-C, 22 p. Available at: https://pubs.usgs.gov/wsp/2001c/report.pdf [Accessed 19 July 2023].

Aron, G., Ball, J.E., Smith, T.A., 1991. Fractal concept used in time of concentration estimates. J. Irrig. Drain. Eng. 117 (5), 635–641.

Asano, Y., Uchida, T., 2018. The roles of channels and hillslopes in rainfall/runoff lag times during intense storms in a steep catchment. Hydrol. Process. 32 (6), 713–728.

Askew, A.J., 1970. Derivation of formulae for variable lag time. J. Hydrol. 10 (3), 225–242.

Azizian, A., 2018. Uncertainty Analysis of Time of Concentration Equations based on First-Order-Analysis (FOA) Method. Am. J. Eng. Appl. Sci. 11 (1), 327–341.

Azizian, A., 2019. Comparison of salt experiments and empirical time of concentration equations. Proc. Inst. Civil Eng. - Water Manage. 172 (3), 109–122.

Bahmanpouri, F., Eltner, A., Barbetta, S., Bertalan, L., and Moramarco, T., 2022. Estimating the average river cross-section velocity by observing only one surface velocity value and calibrating the entropic parameter. Water Resour. Res., 58:10, e2021WR031821.

Bell, F.C., Om Kar, S., 1969. Characteristic response times in design flood estimation. J. Hydrol. 8 (2), 173–196.

Benson, M.A., 1959. Channel-slope factor in Flood Frequency Analysis. J. Hydraul. Div. 85 (4), 1–9.

Beven, K.J., 2020. A history of the concept of time of concentration. Hydrol. Earth Syst. Sci. 24, 2655–2670.

Biggs, B.J.F., Duncan, M.J., Jowett, I.G., Quinn, J.M., Hickey, C.W., Davies-Colley, R.J., Close, M.E., 1990. Ecological characterisation, classification, and modelling of New Zealand rivers: An introduction and synthesis. N. Z. J. Mar. Freshw. Res. 24 (3), 277–304.

Blöschl, G., Sivapalan, M., Wagener, T., Viglione, A., Savenije, H. (Eds.), 2013. Runoff Prediction in Ungauged Basins: Synthesis across Processes, Places and Scales. Cambridge University Press, Cambridge.

Bocchiola, D., De Michele, C., Pecora, S., Rosso, R., 2003. Sul tempo di risposta dei bacini idrografici italiani. L'acqua 7, 45–55 in italian.

Bondelid, T.R., McCuen, R.H., Jackson, T.J., 1982. Sensitivity of SCS models to Curve Number variations. J. Am. Water Resour. Assoc. 18 (1), 111–116.

Bransby Williams, G., 1922. Flood discharges and the dimensions of spillways in India. The Engineer 134, 321.

Brunner, M.I., Seibert, J., Favre, A.-C., 2016. Bivariate return periods and their importance for flood peak and volume estimation. WIREs Water 3 (6), 819–833.

Carter, R., 1961. Magnitude and Frequency of Floods in Suburban Areas. In: Short papers in the geologic and hydrologic sciences, U.S. geol. Survey. Prof. paper 424-B, B9-B11.

Chezy, A. D., 1775. Memoire sur la vitesse de l'eau conduite dans une régole. Reprinted in Annals des Ponts et Chaussées 60, 1921.

Chiu, C.L., and Said, C. A., 1994. Modeling of maximum velocity in open-channel flow. Proc., Hydr. Div. Conf. Vol. 1, New York, N.Y.,I, 381-385.

Chow, V.T., Maidment, D.R., Mays, L.W., 1988. Applied hydrology. McGraw-Hill, New York.

Chow, V.T., 1962. Hydrologic determination of waterway areas for the design of drainage structures in small drainage basins. University of Illinois Engineering Experiment Station Bulletin No. 462, Urbana, IL, 104 p. Available at: https://core.ac.uk/download/pdf/4814575.pdf [Accessed 19 July 2023].

Dooge, J.C.I., 1973. Linear Theory of Hydrologic Systems. Technical Bulletin No. 1468. 1st Edn., Agricultural Research Service, United States Department of Agriculture.

Efstratiadis, A., Koussis, D., Koutsoyiannis, D., Mamassis, N., 2014. Flood design recipes vs. reality: can predictions for ungauged basins be trusted? Nat. Hazards Earth Syst. Sci. 14, 1417–1428.

Espey, W. H. J., Morgan, C.W. and Masch, F.D., 1966. Study of some effects of urbanization on storm runoff from a small watershed. Texas Water Development Board Rep. No. 23. Austin, TX: Center for Research in Water Resources, Univ. of Texas. Available at: https://www.twdb.texas.gov/publications/reports/numbered_reports/doc/R23/R23.pdf [Accessed 19 July 2023].
Flavell, D.J., 1983. The rational method applied to small rural catchments in the south west of western Australia. Civ. Eng. Trans. Inst. Engrs. Aut. 25, 121–127.
Gericke, O.J., 2018. Catchment response time and design rainfall: the key input parameters for design flood estimation in ungauged catchments. J. s. Afr. Inst. Civ. Eng. 60 (4), 51–67.
Gericke, O.J., Smithers, J., 2014. Review of methods used to estimate catchment response time for the purpose of peak discharge estimation. Hydrol. Sci. J. 59 (11), 1935–1971.
Grimaldi, S., Petroselli, A., Tauro, F., Porfiri, M., 2012. Time of concentration: a paradox in modern hydrology. Hydrol. Sci. J. 57 (2), 217–228.
Gupta, V.K., Waymire, E., Wang, C.T., 1980. A representation of an instantaneous unit hydrograph from geomorphology. Water Resour. Res. 16 (5), 855–862.
Hack, J., 1957, Studies of longitudinal stream profiles in Virginia and Maryland, US Geological Survey Professional Paper, 294-B, 1.
Haktanir, T., Sezen, N., 1990. Suitability of two-parameter gamma and three-parameter beta distributions as synthetic unit hydrographs in Anatolia. Hydrol. Sci. J. 35 (2), 167–184.
Hallema, D.W., Moussa, R., Sun, G., McNulty, G., 2016. Surface storm flow prediction on hillslopes based on topography and hydrologic connectivity. Ecol. Process. 5 (13).
California Highways and Public Works. 1955. California culvert practice: Reprint of a series of technical abstracts [appearing in each issue from Aug. 1942 to Sept.-Oct. 1943]. Sacramento, CA: State of California, Dept. of Public Works, Division of Highways.
Hrachowitz, M., Savenije, H.H.G., Blöschl, G., McDonnell, J.J., Sivapalan, M., Pomeroy, J.W., Arheimer, B., Blume, T., Clark, M.P., Ehret, U., Fenicia, F., Freer, J. E., Gelfan, A., Gupta, H.V., Hughes, D.A., Hut, R.W., Montanari, A., Pande, S., Tetzlaff, D., Troch, P.A., Uhlenbrook, S., Wagener, T., Winsemius, H.C., Woods, R.A., Zehe, E., Cudennec, C., 2013. A decade of Predictions in Ungauged Basins (PUB) - a review. Hydrol. Sci. J. 58 (6), 1198–1255. https://doi.org/10.1080/02626667.2013.803183.
Jia, Y., Wang, Z., Zheng, X., Li, Y., 2016. A study on limit velocity and is mechanism and implications for alluvial rivers. Int. J. Sediment Res. 31, 205–211.
Johnstone, D., Cross, W.P., 1949. Elements of applied hydrology. Ronald Press, New York.
Jowett, I.G., 1998. Hydraulic geometry of New Zealand rivers and its use as a preliminary method of habitat assessment. Regul. Rivers: Res. Mgmt. 14 (5), 451–466.
Kennedy, R.J., Watt, W.E., 1967. The relationship between lag time and the physical characteristics of drainage basins in southern Ontario. In: Symposium on Floods and Their Computation. International Association of Scientific Hydrology, Leningrad, pp. 866–874.
Kirpich, Z., 1940. Time of concentration of small agricultural watersheds. Civ. Eng. 10 (6), 362.
Laurenson, E.M., 1962. Hydrograph Synthesis by Runoff Routing. Report No. 66, Water Research Laboratory, University of New South Wales. Available at: https://unsworks.unsw.edu.au/entities/publication/1d112e22-f5a2-452f-8369-d08fbb066f4c [Accessed 19 July 2023].
Leopold, L.B., 1953. Downstream change of velocity in rivers. Am. J. Sci. 251 (8), 606–624.
Leopold, L.B., 1991. Lag times in for small drainage basins. Catena 18 (2), 157–171.
Leopold, L.B. and Maddock, T., 1953. The Hydraulic Geometry of Stream Channels and Some Physiographic Implications. Geological Survey Professional Paper 252. Available at: https://pubs.usgs.gov/pp/0252/report.pdf [Accessed 19 July 2023].
Lighthill, M.J., Whitham, G.B., 1955. On kinematic waves, 1, Flood movement in long rivers. Proc. r. Soc. London, Set. 4, 229, 281–316.
Linsley, R. K., Jr., Kohler, M.A. and Paulhus, J.L.H., 1958. Hydrology for Engineers, McGraw-Hill, 340 pp.
Lopes da Silveira, A.L., 2005. Desempenho de Fórmulas de Tempo de Concentração em Bacias Urbanas e Rurais. Rev. Bras. De Recur. Hídr. 10 (1), 5–23 in Portuguese.
Manning, R., 1889. On the Flow of Water in Open Channels and Pipes. Trans. Inst. Civil Eng. Ireland 20, 161–207.
McCuen, R.H., 2009. Uncertainty analyses of watershed time parameters. J. Hydraul. Eng. 14 (5), 490–498.
McCuen, R.H., Spiess, J.M., 1995. Assessment of kinematic wave time of concentration. J. Hydraul. Eng. 121 (3), 256–266.
McCuen, R.H., Wong, S.L., Rawls, W.J., 1984. Estimating urban time of concentration. J. Hydraulic Eng. Am. Society Civil Engineers 110 (7), 887–904.
McDonnell, J.J., Beven, K., 2014. Debates—The future of hydrological sciences: A (common) path forward? A call to action aimed at understanding velocities, celerities, and residence time distributions of the head water hydrograph, Water Resour. Res. 50 (6), 5342–5350.
McEnroe, B.M. and Zhao, H., 1999. Lag times and peak coefficients for rural watersheds in Kansas. University of Kansas, USA, Report No. K-TRAN: KU-98-1. Available at: https://rosap.ntl.bts.gov/view/dot/14157 [Accessed 19 July 2023].
McEnroe, B.M. and Zhao, H., 2001. Lag times of urban and developing watersheds in Johnson County, Kansas. University of Kansas, USA, Report No. KTRAN: KU–99–5. Available at: https://ntlrepository.blob.core.windows.net/lib/18000/18700/18705/PB2002101487.pdf [Accessed 19 July 2023].
Michailidi, E.M., Antoniadi, S., Koukouvinos, A., Bacchi, B., Efstratiadis, A., 2018. Timing the time of concentration: shedding light on a paradox. Hydrol. Sci. J. 63 (5), 721–740.
Montanari, A., Young, G., Savenije, H.H.G., Hughes, D., Wagener, T., Ren, L.L., Koutsoyiannis, D., Cudennec, C., Toth, E., Grimaldi, S., Blöschl, G., Sivapalan, M., Beven, K., Gupta, H., Hipsey, M., Schaefli, B., Arheimer, B., Boegh, E., Schymanski, S.J., Di Baldassarre, G., Yu, B., Hubert, P., Huang, Y., Schumann, A., Post, D., Srinivasan, V., Harman, C., Thompson, S., Rogger, M., Viglione, A., McMillan, H., Characklis, G., Pang, Z., Belyaev, V., 2013. "Panta Rhei—Everything Flows": Change in hydrology andsociety—The IAHS Scientific Decade 2013–2022. Hydrol. Sci. J. 58 (6), 1256–1275.
Nagy, E.D., Szilagyi, J., Torma, P., 2021. Assessment of dimension-reduction and grouping methods for catchment response time estimation in Hungary. J. Hydrol. Regional Studies 38, 100971.
Natural Environment Research Council (NERC), 1975. Hydrological studies, in Flood studies report, London, England.
Natural Resources Conservation Service (NRCS), 2010. Chapter 15: Time of concentration. In National engineering handbook, Part 630, Hydrology. Washington, DC: Author.
Overton, D.E., Meadows, M.E., 1976. Storm water modeling. Academic Press, New York, USA.
Papadakis, K.N. and Kazan, M.N., 1987. Time of concentration in small rural watersheds. In: Proceedings of the ASCE engineering hydrology symposium, Williamsburg, VA: ASCE, 633–638.
Pezzoli, G., 1970. Schemi e modelli matematici in idrologia. Levrotto e Bella (in italian), Torino.
Pilgrim, D.H., 1977. Isochrones of travel time and distribution of flood storage from a tracer study on a small watershed. Water Resour. Res. 13 (3).
Pilgrim, D.H. and G.E. McDermott, 1982. Design floods for small rural catchments in eastern New South Wales. Civ. Eng. Trans. Inst. Engrs Aust.
Pinto, N.S., Holtz, A.C.T., Martins, J.A., Gomide, F.L.S., 1976. Hidrologia Básica. São Paulo: Edgar Blücher, 278p (in Portuguese).
Putnam, A.L., 1972. Rainfall and runoff in urban areas: a case study of flooding in the Piedmont of North Carolina. In: Proceedings 1957 of the Urban Rainfall Management Problems Conference. University of Kentucky, Lexington, USA, pp. 17–18.
Ramser, C.E., 1927. Run-off from small agricultural areas. J. Agric. Res. 34 (9), 797–823.
Ravazzani, G., Boscarello, L., Cislaghi, A., Mancini, M., 2019. Review of time-of-concentration equations and a new proposal in Italy. J. Hydrol. Eng. 24 (10).
Robinson, J.S., Sivapalan, M., Snell, J.D., 1995. On the relative roles of hillslope processes, channel routing, and network geomorphology in the hydrologic response of natural catchments. Water Resour. Res. 31 (12), 3089–3101.
Rodriguez-Iturbe, I., Valdes, J.B., 1979. The geomorphologic structure of hydrologic response. Water Resour. Res. 15 (6), 1409–1420.
Rossi, F., 1974. Criteri di similitudine idrologica per la stima della portata al colmo di piena corrispondente ad un assegnato periodo di ritorno. In: Atti Del XIV Convegno Di Idraulica e Costruzioni Idrauliche, October. Italy (in italian), Napoli, pp. 10–12.
Rosso, R., 1984. Nash model relation to Horton order ratios. Water Resour. Res. 20 (7), 914–920.
Schaake Jr., J.C., Geyer, J.C., Knapp, J.W., 1967. Experimental examination of the rational method. J. Hydraul. Div. 93 (6).
Schulz, E. F., and Lopez, O. G., 1974. Determination of urban watershed response time. Hydrology Papers, no. 71. Dept. of Civil Engineering, Colorado State University, Fort Collins, Colorado. Available at: https://mountainscholar.org/bitstream/handle/10217/61582/HydrologyPapers_n71.pdf [Accessed 19 July 2023].
Schulz, E.F., 1969. Dimensionless unit hydrographs from tropical watersheds. Presented at the National Fall Meeting American Geophysical Union, December 15-18, San Francisco, California, USA. Available at: https://mountainscholar.org/handle/10217/648 [Accessed 19 July 2023].
Sharifi, S., Hosseini, S.M., 2011. Methodology for identifying the best equations for estimating the time of concentration of watershed sin a particular region. J. Irrig. Drain. Eng. 37 (11), 685–746.
Sheridan, J.M., 1994. Hydrograph time parameters for flatland watersheds. Trans. Am. Soc. Agric. Eng. 37 (1), 103–113.
Sherman, L.K., 1932. Streamflow from Rainfall by the Unit Graph Method. Eng. News Rec. 108, 501–505.
Shook, K.R., Whitfield, P.H., Spence, C., Pomeroy, J.W., 2023. Estimating response times, flow velocities and roughness coefficients of Canadian Prairie basins. Hydrol. Earth Syst. Sci. Discuss. [preprint], doi: 10.5194/hess-2023-51, in review.
Singh, V.P., 1988. Hydrologic systems: rainfall-runoff modeling. Prentice Hall, p. 480.
Singh, P.K., Mishra, S.K., Jain, M.K., 2014. A review of the synthetic unit hydrograph: from the empirical UH to advanced geomorphological methods. Hydrol. Sci. J. 59 (2), 239–261.
Soil Conservation Service (SCS), 1954. National Engineering Handbook, Section 4, Hydrology. Washington, D.C., U.S. Dept. of Agriculture, Soil Conservation Service, 1972. First published as: Hydrology guide for use in watershed planning. Available at: http://irrigationtoolbox.com/NEH/Part%20630%20Hydrology/neh630-ch15.pdf [Accessed 19 July 2023].
Soil Conservation Service (SCS), 1975. Urban Hydrology for Small Watersheds, Technical Release No. 55, Washington, D.C., USA.
Sriwongsitanon, N., Ball, J.E., Cordery, I., 1998. An investigation of the relationship between the flood wave speed and parameters in runoff-routing models. Hydrol. Sci. J. 43 (2), 197–213. https://doi.org/10.1080/02626669809492118.
Taylor, A.B., Schwarz, H.E., 1952. Unit hydrograph lag and peak flow related to basins characteristics. Trans. Am. Geophys. Union 33, 235–246.
Temez, J.R., 1987. Calculo hidrometeorologico de caudales maximos em pequenas cuencas naturales. Madrid: Ministério de Obras Publicasy Urbanismo (MOPU), Direccion General de Carreteras (in spanish).

United States Geological Survey (USGS), 2000. Equations for Estimating Clark Unit-Hydrograph Parameters for Small Rural Watersheds in Illinois. US Dept of the Interior and US Geological Survey, Branch of Information Services, Report 00–4184, Urbana, IL, USA. Available at: https://pubs.usgs.gov/wri/2000/4184/wrir00_4184.pdf [Accessed 19 July 2023].

Watt, W.E., Chow, K.C.A., 1985. A general expression for basin lag lime. Can. J. Civ. Eng. 12, 294–300.

Welle, P.J., and D.E. Woodward, 1986. Time of concentration. Hydrology, Technical Note No. N4. U.S. Department of Agriculture, Soil Conservation Service, NENTC, Chester, PA.

Williams, J. R., 1968. Runoff hydrographs from small Texas Blacklands watersheds. ARS 41-143. USDA-ARS.

Woolhiser, D.A., Liggett, J.A., 1967. Unsteady one-dimensionalflow over a plane - The rising hydrograph. Water Resour. Res. 3 (3), 753–771.

Xia, R., 1997. Relation between mean and maximum velocities in a natural river. J. Hydraul. Eng. 123, 8.