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Objective and process-based catchment classification as a tool for predictions in ungauged basins

HS 29

Evaluation of flow duration curves with assigned return period in heterogeneous basins of Southern Italy

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Introduction

A flow duration curve (FDC) is, by definition, the relationship between any given discharge value and the percentage of time that this discharge is exceeded. It represents the relationship between magnitude and frequency of streamflow discharges.

The procedure proposed in this work is based on the use of distributions supported on a bounded [0,1] interval and exploits the properties of the Complementary Beta (CB) distribution (Jones, 2002). The proposed model, accounts for the interannual variability of the FDC by means of distributions of annual minimum daily discharge and total annual streamflow while the intra-annual variability is described by the CB distribution that, like the Beta distribution, has two parameters whose behaviour characterizes the shape of the FDC and in particular of its tails.

EtaBeta model for a single year

In a generic reference period, consider *Q*(*d*) as the relationship between the observed discharge values and the total time *d*, in which discharge is less than Q(*d*) within the reference period *D*. According to the classical definition of FDC we have

(1) FDC(ǔ)=*Q*(ǔ)

where the quantity ǔ = (D-d) is called ñdurationò of discharge.
Within our probabilistic approach the FDC is considered a probability distribution bounded by the minimum and the maximum observed streamflows, then, *d* may be considered as a measure of *u*, probability of not exceedance of discharge, by means of

(2) $u = \frac{d}{D}$ with 0 Ò*u* Ò1.

Definition of the annual streamflow volume function

Consider the calendar (or water) year as the reference period then, working on daily streamflows, *d* is in days, and D =365 days. In such a case the annual discharge function *Q*(*d*) is provided by ordering the flows in ascending order and the integral function of *Q*(*d*) represents a volume of water, that, for *d* varying from 1 to 365 days, ranges from the minimum daily volume *W*(1) to the total annual volume *W*(*D*).

(3) $W(d) = \int_0^d Q(t)dt$

Derivation of the annual streamflow volume function

Consider the log-transformed variable of *W*

(4) $y = \log(W \mid W > 0)$

and its transformed

(5) $\eta = \left(\frac{y - y_{\min}}{y_{\max} - y_{\min}}\right)$

The variable ñɖò represents a ñstandardized streamflow-volumeò measure, ranging between 0 and 1, with

(6) *ymin* = log(W(1)) and *ymax* = log(W(D))

respectively the logarithms of minimum daily flow and total annual streamflow volume.
Let us assume ɖ is a random variable Beta distributed. Consider parameters *a*>0, *b*>0, and denote by *B*(*a*,*b*), *B*(ɖ,*a*,*b*) and *I*(ɖ,*a*,*b*) = *B* (ɖ,*a*,*b*)/*B*(*a*,*b*) respectively the Beta function, the incomplete Beta function and the incomplete Beta function ratio. The Beta distribution on [0,1] has quantile function

(7) ɖ(*u*)=*FB*-1(*u*)= *I* -1(*u*,*a*,*b*)

where *u* is the probability of not exceedance and *I* -1(*u*,*a*,*b*) is the inverse of the incomplete Beta function ratio.
Then, the incomplete Beta function ratio is fitted to the annual function of ɖ, introduced in equation ( ) and, hence, the stochastic dependence between ɖ and *d* is analyzed, on annual base, considering samples of ɖ and corresponding probabilities *u* = *d/D*, as in equation (2).
Following equations (4) and (5), the annual streamflow volume function *W*(*d*) can be expressed as:

(8) $W(d) = \exp\left[I^{-1}\left(\frac{d}{D}, a, b\right)(y_{\max} - y_{\min}) + y_{\min}\right]$

where ɖ is replaced by *I* -1(*u*,*a*,*b*), as from equation (7), with *u* evaluated as in equation (2).

Derivation of the annual discharge function *Q*(*d*)

Besides Beta distribution, Jones (2002) introduced an alternative continuous distribution on [0,1], the Complementary Beta distribution, let write CB(*a*,*b*), with *a* and *b* parameters, obtained from reversing the roles of distribution and quantile functions of a traditional Beta distribution. Within such approach, the quantities *u* and ɖ, above introduced, may be considered alternatively ñstochastic variableò or ñprobabilityò of each other.
The Complementary Beta distribution has cumulative distribution function

(9) *FC*(*u*)= *I*.-1(*u*,*a*,*b*) = ɖ(*u*)

and density function

(10) $f_C(u,a,b) = \frac{B(a,b)}{\{I^{-1}(u,a,b)\}^{a-1}\{1 - I^{-1}(u,a,b)\}^{b-1}}$

where *u* is considered a random variable and ɖ the probability of not exceedence of *u*. Following equation (3), the annual discharge function may be found from the derivative function of *W*(*d*) as:

(11) $Q(d) = \frac{dW(d)}{dt}$

The derivative in equation (11) may be solved considering *W*(*d*), as from equation (8), with the inverse of the incomplete Beta function ratio replaced by the CB cumulative distribution function *FC*(*u*), as in equation (9), to give

(12) $Q(d) = W(d)(y_{\max} - y_{\min})\frac{dF_C(u)}{du}\frac{du}{dt}$

and finally, replacing the derivative of the cumulative probability function of the CB distribution with its probability density function and *u* = *d/D*, as in equation (2):

(13) $Q(d) = \frac{W(d)}{D}(y_{\max} - y_{\min})f_C\left(\frac{d}{D}, a, b\right)$

where functions *W*(*d*) and *fC*(*u*) are respectively reported in equations (8) and (10).

Annual FDC for fixed T-year return period

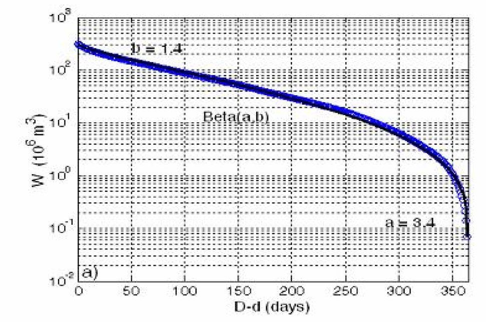
The return period of the minimum annual value of any stochastic variable (observed once or globally in a year) which is ñnot exceededò, on average, once in *T* years, is determined as a function of its probability of not exceedence *p* as *p* = 1/*T*. On the other hand, the return period of the maximum annual value which is ñexceededò, on average, once in *T* years, is determined as a function of the same probability of not exceedence *p* as *p* = 1 ï 1/*T*. Then, in order to represent the overall variability of the streamflow volume function, we preserve the structure of equations (8) and and let *a* and *b* take on the respective observed mean values µ*a* and µ*b*, to write respectively:

(14) $W_p(d) = \exp\left[I^{-1}\left(\frac{d}{D}, \mu_a, \mu_b\right)(y_{\max,p} - y_{\min,p}) + y_{\min,p}\right]$

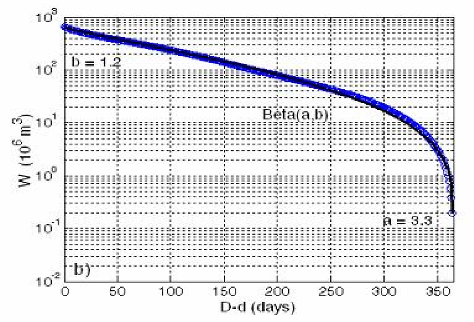
Moreover after a derivation analogous to that from equation (11) to (13), we obtain:

(15) $Q_p(d) = \frac{W_p(d)}{D}(y_{\max,p} - y_{\min,p})f_C\left(\frac{d}{D}, \mu_a, \mu_b\right)$

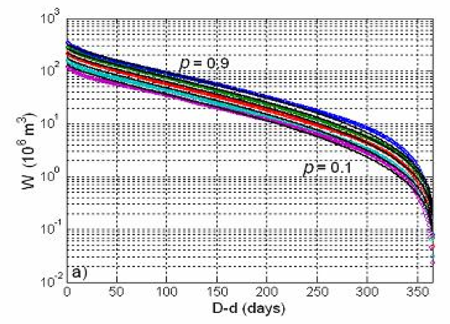
where *ymin*,*p* and *ymax*,*p* are found as corresponding *p*th-quantiles of variables *ymin* and *ymax*, which according to equations (6) are functions respectively of the annual minimum daily streamflow and the total annual streamflow. In equations (14) and (15) we consider their quantiles arising from marginal distributions, and in principle they can be obtained from regional analysis of the respective observations.



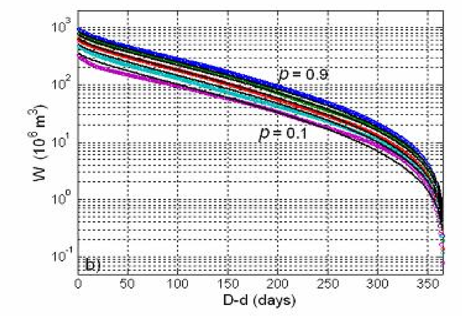
Streamflow volume functions W(D-d) observed (dots) in 1937 and equation (continuous line) including the fitted Beta function ratio for Sinni at Pizzutello.



Streamflow volume functions W(D-d) observed (dots) in 1937 and equation (continuous line) including the fitted Beta function ratio for Sinni at Valsinni.



Annual streamflow volume functions observed (coloured dots) and derived (continuous line) from equation (14) for *p* = 0.1, 0.25, 0.5, 0.75 and 0.9 for Sinni at Pizzutello.



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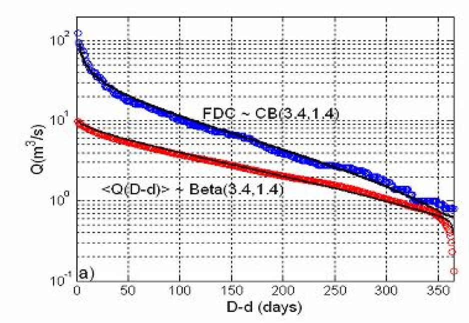
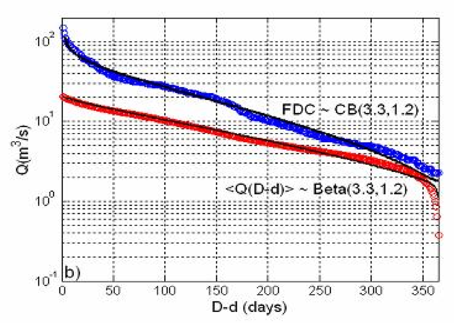


Annual FDCs observed (blue dots) in 1937 and derived as a function of CB(a,b) (continuous line), and corresponding d-average discharge function <Q(D-d)> (red dots), for Sinni at Pizzutello.



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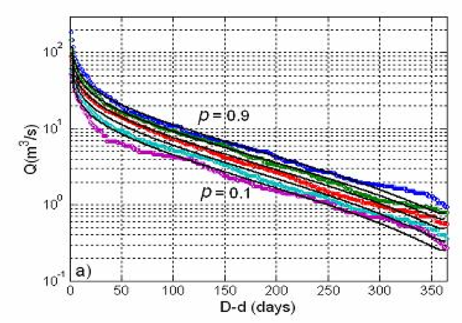
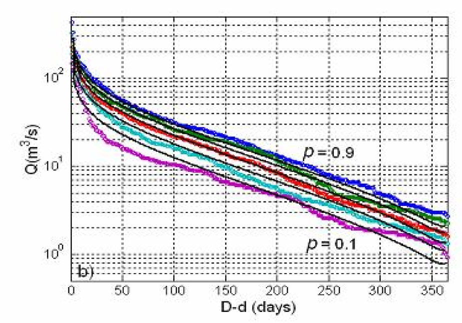


Annual flow duration curves observed (coloured dots) and derived (continuous line) from equation (15) for *p* = 0.1, 0.25, 0.5, 0.75 and 0.9 for Sinni at Pizzutello.



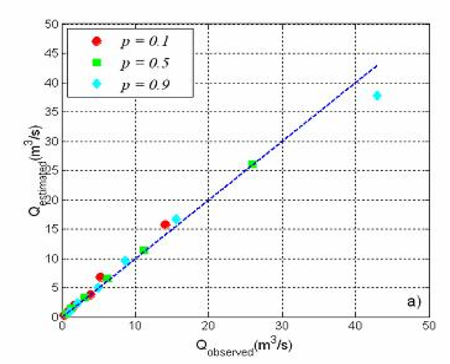
Annual flow duration curves observed (coloured dots) and derived (continuous line) from equation (15) for *p* = 0.1, 0.25, 0.5, 0.75 and 0.9 for Sinni at Valsinni.

STUDY REGION

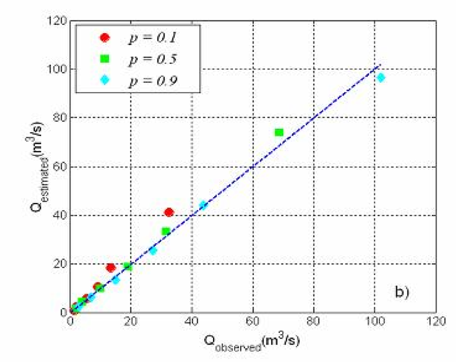
In order to assess the model performance and reliability, two case studies were developed with reference to daily discharge series recorded in two nested sub-basins of the Sinni river, at the gauged stations of Pizzutello and Valsinni, Basilicata, Southern Italy.These discharge series were recorded from 1925 to 1980, almost continuously (see table 1). The calendar year was adopted because the application was more oriented to the analysis of low-flows and volume deficit.

Site	Years of observation	N	N_c	A (Km^2)	µ(Q) (m^3/s)	σ(Q) (m^3/s)	Cv(Q)
Sinni at Pizzutello	1925 – 1928 1930 – 1942 1948 – 1967 1970 - 1980	48	20	232	7.3	13.5	1.9
Sinni at Valsinni	1937 – 1942 1950 - 1976	33	27	1140	20.5	32.9	1.6

Main features of gauged basins and recorded time series.



Estimated *vs* observed values of $Q_p(d)$ for *d* = 7, 182, 347 and *p* = 0.1, 0.5, 0.9, Sinni at Pizzutello.



Estimated *vs* observed values of $Q_p(d)$ for *d* = 7, 182, 347 and *p* = 0.1, 0.5, 0.9, Sinni at Valsinni.

CONCLUSIONS

The annual flow duration curve is considered, in principle, as a distribution function supported on the bounded interval [0,1] and ranging between the annual minimum and maximum daily discharge. The two-parameters Complementary Beta distribution is used to model such annual FDC but, within this rationale, the incomplete Beta function ratio is used in order to fit the integral of the discharge-duration function, which represents the streamflow volume-duration curve. In this way the stochastic overall variability of annual FDCs is modelled analyzing the distribution of four variables: the Complementary Beta parameters *a* and *b*, the annual minimum daily discharge and the total annual streamflow volume. The main advantage of the proposed method lies in its parameterization which is based on the characterization of the minimum annual flow and the total annual streamflow which are variables depending on long term climatic features and are usually well studied and amenable for regional analyses and prediction. In particular results reported in this paper show that the inter-annual variability of the FDC, whose prediction allows to assign a return time to the whole FDC, is completely represented by them. On the other hand the two variables *a* and *b*, parameters of the Complementary Beta distribution, completely characterize the intra-annual variability of the FDC. They are responsible of the shape of the two tails, in particular *a*, characterizes the low-flow part of the FDC which depends on groundwater\subsurface contribution, while *b* characterizes the high-flow part of the FDC and is more affected by climate and precipitation features. The analysis of time series of *a* and *b* shows a significant (and expected) dependency on each other and on other climatic factors, nevertheless their inter-annual variability seems to be less important and their fractiles for assigned probability Φ are replaced by respective expected values μ_a and μ_b without significatively affect the prediction of significant $Q_{d,\Phi}$ and $W_{d,\Phi}$ values. Such result is of particular interest since it opens a field of investigation about the spatial variability of μ_a and μ_b and their dependency on measurable geomorphoclimatic quantities. Thus the EtaBeta model offers interesting perspectives in terms of regionalization and prediction in ungauged basins. It is also useful to highlight that the EtaBeta model also allows to account, within a unique framework, for different characteristic basin functions useful for design and management of structural and non-structural water resources systems from the definition of environmental minimum streamflow requirements to the optimization of stream diversion and regulation structures. Besides FDC, we refer to minimum annual flow distribution, maximum annual flow distribution and total annual flow distribution and other related quantities like, for example, the volume W_M of water diverted in a time interval D by an hydraulic structure with Q_M maximum flow discharge.

REFERENCES

Castellarin, A., Vogel, R.M., Brath, A., (2004a) A stochastic index flow model of flow duration curves, Water Resour. Res., 40, (3), W03104, doi:10.1029/2003WR002524.

Castellarin A., G. Galeati, L. Brandimarte, A. Montanari, and A Brath, (2004b), Regional flow-duration curves: reliability for ungauged basins, *Adv Water Res.* 27 (10): 953-965.

Claps, P., Fiorentino, M., (1997) Probabilistic Flow Duration Curves for use in Environmental Planning and Management, in Harmancioglu et al. (eds.) Integrated Approach to Environmental Data Management Systems, NATO-ASI series 2 (31), pp. 255-266, Kluwer, Dordrecht, The Netherlands

Hosking, J. R. M., (1990) L-moments: Analysis and estimation of distributions using linear combinations of order statistics, *J. R. Stat. Soc.*, Ser. B, 52, 105-124.

Hosking, J. R. M., and J. R. Wallis (1997), Regional Frequency Analysis: An Approach Based on L-Moments, Cambridge Univ. Press, New York.

Iacobellis, V., A. F. Piccinni and C. Giordano, (2004) Probabilistic evaluation of flow duration curves in semi-arid basins, Proc. of the Ninth International Symposium on river Sedimentation (9th ISRS), Yichang, China, 18-21 Oct.

Jones, M.C., (2002) The complementary beta distribution, Journal of Statistical Planning and Inference 104 (2): 329-337.

Lane, P.N.J., A.E. Best, K. Hickel, and L. Zhang, The response of flow duration curves to afforestation, *J. Hydrol.*, 310, pp 253-265, 2005.

Maidment, chap. 8, pp. 8-27ï 8-28, McGraw-Hill, New York, 1993.

Mosley M.P., and A.I. McKerchar, (1993) Streamflow, in Handbook of Hydrology, edited by D. R.

Singh, K.P., and J.B. Stall, (1974), Hydrology of 7-day 10-yr low flow, *J. Hydraulic Division*, ASCE, 100, 1753.

Singh R.D., S.K. Mishra, and H. Chowdhary, (2001), Regional flow-duration models for large number of ungauged Himalayan catchments for planning microhydro projects, *J. of Hydrolog. Eng.*, ASCE, Vol. 6, N. 4

Smakhtin, V. U. (2001), Low flow hydrology: A review, J. Hydrol., 240, 147ï 186.

Vogel, R. M., and N. M. Fenessey, (1994), Flow-duration. I: New interpretation and confidence intervals, *J. Wat. Resour. Plann. and Manage.*, 120(4), 485-504.