CONCEPTUAL BASIS OF STOCHASTIC MODELS OF MONTHLY STREAMFLOWS

P. CLAPS
Dept. Environm. Eng. & Phys.
University of Basilicata
Potenza
Italy

1. Summary

Annual and monthly streamflow time series are mostly modelled using linear models of the ARMA class. Model identification is usually performed through statistical procedures (e.g. Noakes et al., 1985) or, sometimes, by describing the processes as linear conceptual models whose equations can be rearranged in order to assume AR or ARMA representation (e.g. Salas and Smith (1981), Moss and Bryson (1974).

The aim of the present work is to reproduce streamflow time series at both annual and monthly scales by mean of ARMA models whose order is identified through simple conceptual models of the processes. Explicit correspondences between conceptual and stochastic parameters result from the identification procedure.

2. Conceptual Model of Annual Streamflows

Efficient simulation of the annual streamflow process is provided by the Thomas-Fiering conceptual model. In this model the precipitation x_t in the year t is assumed to reach the stream in the amount $(1-c_1-c_2)x_t$ (surface runoff) where c_1x_t is the amount of infiltration and c_2x_t is the amount of evaporation. The annual streamflow is the sum of the surface runoff and of the groundwater contribution c_3V_{t-1}, where V_t is the storage volume at the end of the year t.

This conceptual model has been demonstrated (Salas and Smith, 1981) to lead to an ARMA(1,1) stochastic model, which seems to be the most adequate to describe the annual streamflow process.

However, this model, in the form derived in the above mentioned paper, does not provide explicit relationships between conceptual and stochastic parameters.

In order to reconcile this lack of agreement, a modification of the conceptual model with regard to the system input is proposed. The watershed is supposed to be fed by the *effective* rainfall rather than by the *total* rainfall. In such a way, the evapotranspiration process disappears from the mass balance equations, reducing the conceptual parameters to the number of two.

The streamflow and the groundwater storage mass-balance equations are

$$D_t = c_K V_{t-1} + a(1-r_K)I_t + (1-a)I_t$$

$$V_t = (1-c_K) V_{t-1} + a r_K I_t$$

In the above equations, D_t is the streamflow, V_t is the storage volume I_t is the effective rainfall (or net input) and $(1-c_K)$ is the *recessio coefficient of the storage volume.*

Conceptual parameters are: the infiltration coefficient a, and th storage coefficient K ($K = -[\ln(1-c_K)]^{-1}$).

The feasibility of accounting for the within-year effective rainfal distribution is also added through a predetermined coefficient r_K, whic is called the *recession coefficient of the within-period infiltration* an represents the rate of the infiltration volume reaching the stream at th end of the year. The coefficient r_K depends on the aquifer storag coefficient K and on the shape of the within-year effective rainfall curv (see Moss and Bryson (1974) for the case of concentrated input).

If an analytical representation is given to the effective rainfall curv i(t), the r_K - K relationship can be obtained by solving the expression:

$$(1-r_K) \int_0^1 i(z)dz = \int_0^1 e^{-z/K} / K \cdot \int_0^z i(m)e^{-m/K} \, dm \, dz$$

The proposed conceptual model is shown in Figure 1.

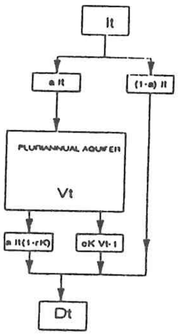

Figure 1. Conceptual model of annual flows.

3. Annual Flows: Relationships Between Conceptual and Stochastic Parameters

By combining the above mass balance equations, the following ARMA(1,1) model is obtained with parameters a and K:

$$D_t - (1 - c_K) D_{t-1} = (1-a\ r_K) I_t - [(1 - c_K) - a\ r_K] I_{t-1}$$

On the other hand, a general ARMA(1,1) stochastic model can be written as:

$$D_t - \Phi D_{t-1} = I^*_t - \theta I^*_{t-1}$$

with parameters: Φ = autoregression coefficient, θ = moving average coefficient. The parameters a and K can be expressed in terms of Φ and θ as follows:

$$a = (\Phi - \theta) / (1 - \theta) \qquad K = -[\ln (\Phi)]^{-1}$$

and the relationship between the residual I^*_t and the net input (effective rainfall) I_t is:

$$I^*_t = I_t (1 - a r_K)$$

4. Conceptual Model of Monthly Flows

In Italian basins with two distinct climatic seasons and no snow melt runoff, the presence of at least two different groundwater contributions can be observed during the recession phase. The first one is due to the so-called *deep aquifer* and its presence is evident at the end of the dry season. It has a pluriannual decay, which implies that the storage coefficient is greater than one year.

The second one is due to the so-called *seasonal aquifer*, which can be observed at the end of the rainfall season, where an exponential decay occurs below flood peaks. This aquifer has a seasonal response, because it fills and empties within the year.

Those considerations suggest a conceptual model with two linear reservoirs in parallel plus a diversion, with no lag, representing the direct runoff (Fig. 2).

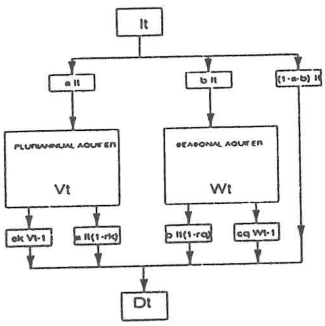

Figure 2. Scheme of the proposed conceptual model of monthly flows.

The model described therefore has four conceptual parameters: two storage coefficients k and q (respectively for the deep and for the seasonal aquifer) both expressed in months, and the two infiltration coefficients a (which is the same as for the conceptual model of annual streamflows) and b.

5. Monthly Flows: Relationships Between Conceptual and Stochastic Parameters

Streamflow and groundwater storage mass balance equations can be written as:

$$D_t = c_k V_{t-1} + a(1-r_k)I_t + c_q W_{t-1} + b(1-r_q)I_t + (1-a-b)I_t$$

$$V_t = (1-c_k) V_{t-1} + a r_k I_t$$

$$W_t = (1-c_q) W_{t-1} + b r_q I_t$$

With regard to the seasonal aquifer, W_t is the storage volume and $(1-c_q)$ is the recession coefficient of the storage volume.

Rearranging, an ARMA(2,2) stochastic model is obtained:

$$D_t - \Phi_1 D_{t-1} - \Phi_2 D_{t-2} = I^*_t - \Theta_1 I^*_{t-1} - \Theta_2 I^*_{t-2}$$

The coefficients of the above canonical representation have conceptual expressions:

$$\Phi_1 = (1-c_k) + (1-c_q) \qquad \Theta_1, \Theta_2 = f(a,b,c_q,c_k,r_q,r_k)$$

$$\Phi_2 = -(1-c_k) \cdot (1-c_q)$$

And the residual - net input relationship is:

$$I^*_t = I_t (1 - a r_k - br_q)$$

Given that the net input is a seasonal process, the stochastic model is an ARMA(2,2) with pseudo-periodic residual.

6. Estimation of ARMA(2,2) Model Parameters

The model was applied to streamflow time series recorded in Italy.

Parameters estimation, performed with classical methods, did not give satisfactory results in our judgment. There are two reasons for this.

First, the storage coefficient k of the deep aquifer assumes very high values with respect to the time scale. In fact, the acceptance zone of the stochastic AR parameters is determined by the constraints:

$$\Phi_2 + \Phi_1 < 1 \; ; \; \Phi_2 - \Phi_1 < 1 \; ; \; -1 < \Phi_2 < 1$$

and when k increases, the coefficient $(1-c_k)$ approaches unity and the sum $\Phi_2 + \Phi_1$ moves toward the limit value 1.

Second, the seasonal aquifer runoff and the total runoff have similar periodicities. For that reason, any deseasonalisation of the streamflow series tends to eliminate the presence of the seasonal aquifer contribution.

The aforementioned drawbacks seem to prove the impossibility of simultaneously estimating the parameters of the two aquifers.

A way to overcome the former problem is to estimate deep and seasonal aquifer parameters independently, using separate models in two different time scales.

The latter point suggests not deseasonalizing the time series under consideration before the estimation stage.

7. Two-stage Model of Monthly Flows

A combined model is proposed which takes advantage of the conceptual and stochastic features of the streamflow process at both annual and monthly time scales.

At the annual time scale, pluriannual aquifer parameters and effective annual rainfall series are estimated.

Then, the deep aquifer runoff is calculated at the monthly time scale. The estimation is performed by disaggregating the series of annual estimated effective input and putting the resulting monthly series into the deep aquifer component. The disaggregation is actually a 'seasonalization' of the mean annual net input and consists of constraining the net input monthly series to have the same periodicity as the monthly flow series.

At that time, a decomposition of the flow is possible at the monthly scale, in order to separate the flow consisting of direct and seasonal aquifer components (which can be considered a 'subprocess' D'_t of the process D_t) from the deep aquifer flow (see Fig. 3).

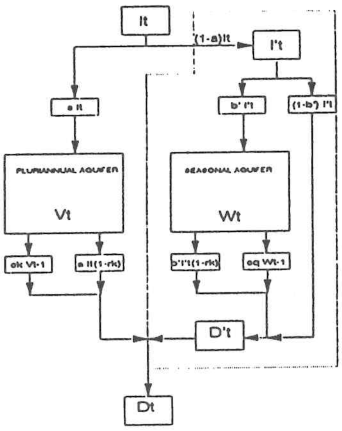

Figure 3. Scheme of the proposed two-stage conceptual model of monthly flows.

The conceptual model of the "direct plus seasonal aquifer" flow leads to an ARMA(1,1) stochastic model with pseudo-periodic residual that arises from the equation of a system composed by one linear reservoir plus diversion with no lag, as in the case of annual streamflows, fed by the pseudo-periodic process $(1-a)I_t$.

The model estimation structure consists of the following steps:

TWO-STAGE MODEL OF MONTHLY FLOWS: ESTIMATION PROCEDURE
ESTIMATION OF THE DEEP AQUIFER CONCEPTUAL PARAMETERS VIA STOCHASTIC MODEL OF ANNUAL FLOWS
DISAGGREGATION OF THE ANNUAL EFFECTIVE INPUT UNDER THE PERIODICITY HYPOTHESIS
EVALUATION OF THE DEEP AQUIFER MONTHLY RUNOFF ORIGINATED BY THE ABOVE INPUT
CALCULATION OF THE 'SUBPROCESS' SERIES D'_t
ESTIMATION OF THE SEASONAL AQUIFER CONCEPTUAL PARAMETERS VIA STOCHASTIC MODEL OF THE 'SUBPROCESS' FLOWS D'_t
EVALUATION OF THE EFFECTIVE INPUT MONTHLY SERIES I_t
EVALUATION OF THE DEEP AND SEASONAL AQUIFER RUNOFFS FROM THE INPUT I_t

8. Applications and Conclusions

The combined model described has been applied with satisfactory results to some river time series in southern Italy. As shown in Figure 4, deep and seasonal aquifer flow reconstruction seems reasonable, as well as the values of conceptual parameters. Nevertheless, further studies are needed in order to improve estimation efficiency, which is stressed by the periodicity of the residuals.

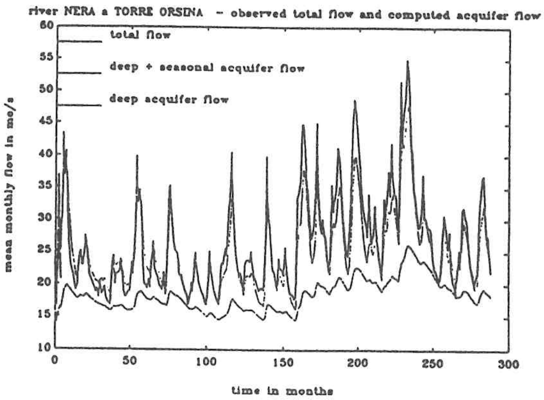

Figure 4. Reconstruction of the deep and seasonal aquifer runoffs. The conceptual parameters, calculated from stochastic parameter estimates, are: a=0.71, K=4.13 years, b=0.252, q=2.93 months.

9. References

Moss, M.E. and Bryson, M.C. (1974) 'Autocorrelation structure of monthly streamflows', Water Resources Research, 10(4), pp 737-744.

Noakes, D.J., McLeod, A.I. and Hipel, K.W. (1985) 'Forecasting monthly riverflow time series', Int. J. Forecast. 1, pp 179-190.

Salas, J.D. and Smith, R.A. (1981) 'Physical basis of stochastic models of annual flows', Water Resources Research, 17(2), pp 428-430.