Uncertainty compliant design-flood estimation

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Abstract.

Hydraulic infrastructures are commonly designed with reference to target values of flood peak, estimated using probabilistic techniques, such as flood frequency analysis. The application of these techniques underlies levels of uncertainty, which are sometimes quantified but normally not accounted for explicitly in the decision regarding design discharges. The present approach aims at defining a procedure which enables the definition of UNcertainty COmpliant DEsign (UNCODE) values of flood peaks. To pursue this goal, we first demonstrate the equivalence of the Standard design based on the return period and the cost-benefit procedure, when linear cost and damage functions are used. We then use this result to assign an expected cost to estimation errors, thus setting a framework to obtain a design flood estimator which minimises the total expected cost. This procedure properly accounts for the uncertainty which is inherent in the frequency curve estimation. Applications of the UNCODE procedure to real cases leads to remarkable displacement of the design flood from the Standard values. UNCODE estimates are systematically larger than the Standard ones, with substantial differences (up to 55%) when large return periods or short data samples are considered.

1. Introduction

The practical objective of a flood frequency analysis is to obtain, for a given return period T, a design flood, which is generally represented by the quantile of a flood frequency curve corresponding to a particular T. The specific mathematical representation of the flood frequency curve can be obtained either using locally available data samples, or from regional flood frequency analysis. The application of these techniques underlies levels of uncertainty which have recently received increasing attention in the scientific literature: for example, *De Michele and Rosso* [2001],*Cameron* [2000], *Brath et al.* [2006], *Blazkova and Beven* [2009] *Laio et al.* [2011], *Liang et al.* [2012] and *Viglione et al.* [2013] have attained a convincing quantification of the uncertainty involved in the statistical estimation of the flood frequency curve. ***In the United States, the U.S. Arms Corps of Engineers (USACE) has been putting a lot of effort for more than 20 years, since the beginning of '90s in developing uncertainty - compliant comprehensive design flood procedure for the United States of America, as reported in U.S. Army Corps of Engineers(USACE) [1996]. There, the uncertainty implied in each step of the design flood procedure is accounted for. However, as highlighted in Davis et al. [2008], the USACE procedure do not provide decisional criteria to follow in uncertainty conditions: uncertainty has to be taken into account but no rules are provided to converge to final design values.*** Uncertainty can be quantified in terms of quantile standard deviation, or in terms of the full probability distribution of the quantile. In the case of flood frequency analysis this means that, for a given return period T, a probability distribution function of the (single) design flood estimator can be provided. In most cases, results of the uncertainty analysis are provided in terms of a ”reference” frequency curve associated with its confidence bands (see Figure 1).

Whatever the approach used to define a flood quantile estimator, the statistical inference will be affected by uncertainty that have both epistemic and aleatory nature[e.g., *Bodo and Unny*, 1976; *Merz and Thieken*, 2005]. While the latter cannot be tackled, because it refers to the natural variability of the events under study, the former depends on the amount of available data and on capacity of the inference procedure to reproduce the underlying hydrological processes. The most relevant sources of epistemic uncertainties are data availability and model selection. In a regional statistical analysis, uneven data sets produce effects that have been studied [e.g., *Stedinger and Tasker*, 1985; *Reis et al.*, 2005] in terms of performance of the statistical procedure when a regional statistical analysis is performed. Accuracy and robustness of the regional estimates can be assumed and inference procedures can be adapted by properly weighting the initial data. Model selection is also a limiting factor, mainly concerned with: i) the choice of the probability distribution function and ii) the choice of the parameters estimation technique. Regarding point i), different families of probability distribution functions are available and there is a great amount of subjectivity in the selection of the best distribution to be adopted. This subjectivity is critical, because, using the same data, different probability distribution functions can produce quite different design values for large return periods [see e.g., *Laio et al.*, 2011], even though, for low return periods, the obtained fitting is good for all distribution functions[*Laio et al.*, 2009]. With regard to point ii), the uncertainty deriving from the specific parameter estimation technique is generally dependent on the bias and variance of the estimators [for a more detailed analysis see *Tung and Yen* [2005] and references therein].

Under this prospective, the definition of "The" design flood probability distribution function for a given return period appears to be the result of several 'averaging' procedures, not necessarily producing the most meaningful result. From this consideration, the main question and motivation behind this paper arises: can a reasonable design flood estimator be devised for a probability distribution function associated with its measurable uncertainty?

To address this question, ***a model in which standard methods for flood frequency analysis are casted in a cost-benefit analysis decision framework is proposed.*** ~~a model in which probabilistic design is casted in a cost73 benefit analysis decision framework is proposed~~ In this sense, the present paper shares a similar scientific background with a recent paper by *Su and Tung* [2013]. However, *Su and Tung* [2013] concentrate their attention on the verification rather than design of hydraulic infrastructures; moreover, they extend their analysis to different risk-based decision-making criteria, which is not necessary here thanks to the relation between cost-benefit analysis and standard flood frequency analysis established in section 2.2.

The conceptual bases of the cost-benefit approach procedure in its traditional form (without uncertainty) are presented in section 2.1 and relations between standard flood frequency analysis and cost - benefit analysis are defined in section 2.2. The application of cost-benefit approach to flood frequency analysis in uncertain conditions is then described in section 3. The whole model is hence applied in section 5 to an extensive data set of annual flow peaks from North-Western Italy basins; results are finally discussed in the conclusion section.

2. The Least Total Expected Cost approach to design (without uncertainty)

2.1. Main features of a cost-benefit analysis

The cost-benefit approach is not frequently used in practice for the design of hydraulic infrastructures, even though some applications are available in the literature [*Tung and Mays*,1981; *Ganoulis*,2003 and *Jonkman*,2004]. In general, given x^* a decision variable, the purpose of a cost-benefit analysis is to obtain the optimal value of the decision variable, x^*_{opt}, comparing costs and benefits each choice of x^* implies. In the case of hydraulic infrastructures the decision variable x^* is usually the design flood q^*. The optimal design flood estimator q^*_{opt} can be obtained by quantifying and comparing costs and damages related to different design floods. The above-mentioned comparison can be performed using the Least Total Expected Cost approach (LTEC) to design [*Bao et al.*, 1987]. LTEC application requires the definition of the cost function, $CF\left(q^* \mid \mathbb{C}\right)$, which measures costs related to different design flood values q^*, e.g. referred to the initial construction and to the maintenance phases. ~~Costs are assumed to increase proportionally to the design flood q^* and are equal to 0 when $q^* = 0$.~~ The relationship between cost and q^* is parametrised according to the type of function considered (e.g., linear, parabolic, etc.) and to a vector of parameters $\mathbb{C}$. For instance, a general linear cost function is given by

$$CF\left(q^* \mid \mathbb{C}\right) = c_0 + c \cdot q^* \tag{1}$$

where c_0 (the y-intercept) and c (the slope) are parameters. Figure 2b depicts an example of a cost function (linear, solid line). ***Costs are assumed to increase proportionally***

to the design flood q^* and are equal to 0 when $q^* = 0$. Figure 2b depicts the example of the linear cost (linear, solid line) with intercept equal to 0 as assumed in the paper.

The damage function $DF(q^*, q \mid \mathbb{D})$ measures the expenses needed to recover from a flooding when a discharge q greater than the design value q^* occurs. *Stedinger* [1997] encourages the use of the expected damage function for hydraulic design purposes [see also *Goldman* [1997]], but so far no clear consensus exists [see *Davis et al.*, 1972; *Beard*, 1990; *Beard*, 1997 and *Beard*, 1998] about the efficiency of the expected damage probability to obtain flood estimators.

However, models for flood damage evaluation have recently benefited from a great effort of research [e.g. *Merz and Thieken*, 2009; *Merz et al.*, 2010; *Vogel and Scherbaum*, 2012; *Merz et al.*, 2013 and *Vogel and Merz*, 2013]. In very general terms, damage functions can be related to the discharge q by means of a function with a threshold:

$$DF(q^*, q \mid \mathbb{D}) = \begin{cases} \Delta\left(q^*, q \mid \mathbb{D}\right) & \text{if } q > q^* \\ 0 & \text{if } q \leq q^* \end{cases} . \tag{2}$$

In equation 2 the function Δ depends on the design flood q^*, on the discharge q of the flooding event and on a vector of parameters $\mathbb{D}$ associated to the type of the function Δ (e.g., linear, parabolic, etc.). To exemplify, Figure 2a depicts a piecewise linear damage function,

$$\Delta\left(q^*, q \mid \mathbb{D}\right) = d_0 + d \cdot \left(q - q^*\right), \tag{3}$$

where d_0 and d are parameters. If $q > q^*$, the damage increases proportionally to the amount of the discharge excess $q - q^*$. Both the design flood estimator q^* and the actual discharge q are random variables. In order to calculate the Expected Damage (ED) corresponding to a design flood it is necessary to apply the Expected Value operator, e.g. the integral over the whole domain of the random variable q of the damage function $\Delta(q^*, q \mid \mathbb{D})$ multiplied by the flood probability distribution function $p(q \mid \boldsymbol{\Theta})$ (where $\boldsymbol{\Theta}$ is the set of parameters of the probability distribution function). The relation is:

$$ED(q^* \mid \mathbb{D}, \boldsymbol{\Theta}) = \int_{q^*}^{\infty} \Delta(q^*, q \mid \mathbb{D}) \cdot p(q \mid \boldsymbol{\Theta})\, \mathrm{d}q. \tag{4}$$

Note that the domain of integration starts at the value q^* because the damage function is equal to 0 for values lower than q^*. The Expected Damage function $ED(q^* \mid \mathbb{D}, \boldsymbol{\Theta})$, as depicted in Figure 2b, is therefore a function of q^* and allows one to define the optimal design discharge q^*_{opt}. The latter comes from summing up construction costs CF and Expected Damage (which of course decreases with the increasing of the security level related to q^*) and searching for a minimum of the Total Expected Cost (C_{TOT}, fig. 2b). Therefore, the Total Expected Cost function can be defined as:

$$C_{TOT}(q^* \mid \mathbb{C}, \mathbb{D}, \boldsymbol{\Theta}) = CF(q^* \mid \mathbb{C}) + \int_{q^*}^{\infty} \Delta(q^*, q \mid \mathbb{D}) \cdot p(q \mid \boldsymbol{\Theta})\, \mathrm{d}q. \tag{5}$$

Searching for the minimum of C_{TOT} allows one to select the optimal design flood estimator as

$$q^*_{opt} = \underset{q^*}{\operatorname{argmin}} \quad [C_{TOT}(q^* \mid \mathbb{C}, \mathbb{D}, \boldsymbol{\Theta})]. \tag{6}$$

2.2. Relations between flood frequency analysis and cost-benefit analysis

Once q^*_{opt} is obtained, it is interesting to compare this value with the design flood value q_T obtained from standard flood frequency analysis. When a return period T is set, this is equivalent to setting a non-exceedance probability $1 - \frac{1}{T}$ for the design flood and calculating the corresponding quantile,

$$q_T = P_q^{-1}\left(1 - \frac{1}{T}\middle|\mathbf{\Theta}\right), \tag{7}$$

where P_q is the the cumulative distribution function and P_q^{-1} is its inverse, i.e. the quantile function.

On the other hand, q^*_{opt} derived from LTEC depends on $DF(q^*, q \mid \mathbb{D})$ and $CF\left(q^* \mid \mathbb{C}\right)$. If linear functions are used for both terms, as in equations (3) and (1), q^*_{opt} from the LTEC procedure comes to be equal to q_T based only on the condition $\frac{d}{c} = T$, where d and c are defined in equations (3) and (1). This equivalence can be analytically demonstrated by rewriting equation (5) using piecewise linear cost and damage functions as follows:

$$C_{TOT}\left(q^* \mid c, d, \mathbf{\Theta}\right) = c \cdot q^* + \int_{q^*}^{\infty} d \cdot \left(q - q^*\right) \cdot p\left(q \mid \mathbf{\Theta}\right) \mathrm{d}q. \tag{8}$$

Taking the derivative of the total expected cost function with respect to q^* and setting it to 0, one obtains

$$\frac{d}{c} = \frac{1}{1 - P_q(q^* \mid \mathbf{\Theta})} = T. \tag{9}$$

In other words, designing an hydraulic infrastructure with a return period T is analytically equivalent to applying a cost-benefit approach with linear cost and damage functions and with $d = c \cdot T$. Validity of equations (8) and (9) can be recognized considering that the linear functions (3) and (1) can be seen as the result of expanding more complicated cost and damage functions in a Taylor series, and truncating these expansions to the first order. Suppose to fix the value of T: once the return period is set, the slope of the damage function is implicitly assumed to be $T-$times larger than the slope of the cost function, $d = c \cdot T$ ***, because*** $\frac{d}{c} = T$ ***acts as a magnifying factor of damage vs***

cost. This means that: i) this condition can be applied even if the actual costs of the infrastructure are unknown; ii) the global number of parameters of equation (8) is exactly the same as that of the traditional flood frequency analysis. This means that the application of the linear cost - benefit model itself does not introduced further sources of uncertainty to the traditional inference procedure, i.e. flood frequency analysis. Lot of effort has recently been put in developing damage models and risk analysis procedure which are essential to calculate, let say, "real" damage functions. These functions are usually non-linear and their parameters are calibrated on past flood scenarios or synthetic flood scenarios. Normally, te shape of the damage function plays a role (Arnell [1989]) the uncertainty associated to these functions is high (Merz and Thieken, 2009, Apel, 2010). The application of non-linear damage functions would introduce more parameters in the model, and, above all, more uncertainty. Though simple, the proposed linear model has the value of being exactly an equivalent formulation of the traditional flood frequency analysis.

~~this condition can be applied even if the actual costs of the infrastructure are unknown.~~

Once this simplified, yet complete, LTEC procedure to obtain q^*_{opt} is set, we can take into account the effects of parametric uncertainty on a LTEC procedure. This is described in the following section.

3. The Least total expected cost approach to design with uncertainty

Probability distribution functions $p(q \mid \mathbf{\Theta})$ of flood peaks describe the quantiles of a random variable q based on a set of parameters $\mathbf{\Theta}$ that are estimated according to a best-fit criterion which adapts the cumulative probability function to the sample cumulative frequencies. Parameter estimates are themselves random variables: therefore the estimated values are uncertain and this uncertainty propagates to the whole flood frequency curve.

When considering parameters $\boldsymbol{\Theta}$ as random variables, a framework is needed to account for uncertainty in the definition of the flood quantile q_T. One of the techniques aiming at accounting for this uncertainty is the Bayesian approach, first introduced in statistical hydrology by *Wood and Rodriguez-Iturbe* [1975] and *Stedinger* [1983]. In the Bayesian approach the pdf $p(q \mid \boldsymbol{\Theta})$ is multiplied by the parameters pdf $h(\boldsymbol{\Theta})$ and integrated in the parameter space, according to the Total Probability theorem [*Kuczera*, 1999], as follows:

$$\widetilde{p}(q) = \int_{\boldsymbol{\Theta}} p\left(q \mid \boldsymbol{\Theta}\right) \cdot h\left(\boldsymbol{\Theta}\right) \mathrm{d}\boldsymbol{\Theta}. \tag{10}$$

Stedinger [1983] called $\widetilde{p}(q)$ the design flood distribution or design flood expected probability [see also *Kuczera* [1999] and references therein]. The parameter distribution function $h(\boldsymbol{\Theta})$ describes how precisely the estimates of parameters are known [*Kuczera*, 1999]. As the number of parameter of the set $\boldsymbol{\Theta}$ is usually more than 1, the distribution $h(\boldsymbol{\Theta})$ is generally a multivariate function.

Substituting in equation (8) $p\left(q \mid \boldsymbol{\Theta}\right)$ with the design flood distribution of equation (10) one obtains

$$\begin{aligned}\widetilde{C}_{TOT}\left(q^* \mid c, d\right) &= c \cdot q^* + \int_{q^*}^{\infty} d \cdot \left(q - q^*\right) \cdot \widetilde{p}(q) \mathrm{d}q = \\ &= c \cdot q^* + \int_{q^*}^{\infty} d \cdot \left(q - q^*\right) \cdot \int_{\boldsymbol{\Theta}} p\left(q \mid \boldsymbol{\Theta}\right) \cdot h\left(\boldsymbol{\Theta}\right) \mathrm{d}\boldsymbol{\Theta} \mathrm{d}q,\end{aligned} \tag{11}$$

where $\widetilde{C}_{TOT}\left(q^* \mid c, d\right)$ is the Total Linear Expected Cost function in uncertain conditions.

As this paper is focused on at-site flood frequency analysis, the a-prior contribution in the definition of the Bayes' rule is neglected. However, there is no restriction in taking into account this contribution, which is highly recomended when avaiable (see Stedinger [1997], Kuczera [1999]), both in equation (10) and equation (11).

The estimator corresponding to the minimum of $\widetilde{C}_{TOT}\left(q^* \mid c, d\right)$ is the UNcertainty COmpliant DEsign (UNCODE) flood estimator which will be called q^*_{unc} in the following:

$$q^*_{unc} = \underset{q^*}{\operatorname{argmin}} \quad \widetilde{C}_{TOT}\left(q^* \mid c, d\right). \tag{12}$$

It is important to remark that, ***when introducing parameter uncertainty to the linear model, the relationship of equation (9) still holds. This implies that the global number of parameters of equation (11) remains the same as that of equation (8) (LTEC without uncertainty, see section 2.2).***

~~using the relationship $\frac{d}{c} = T$, the global number of parameters of equation (11) remains the same as that of equation (8) (LTEC without uncertainty);~~ In fact, c acts as a scaling factor for the total cost and does not affect the position of the minimum. Therefore, differences between the design flood estimators q^*_{unc} and $q_T = q^*_{opt}$ can be fully ascribed to consideration related to parametric uncertainty.

4. Model implementation

4.1. Numerical instances

This section is devoted to describe how the symbolic model of equation (11) can be implemented in practice to obtain optimal design flood estimators q^*_{unc} under uncertainty.

First of all, the integral of the design flood distribution in equation (10) must be solved. An analytical solution of the Expected Probability model of equation (10) exists only if $p\,(q \mid \boldsymbol{\Theta})$ is a 2-parameter Lognormal distribution [*Wood and Rodriguez-Iturbe*, 1975 and *Stedinger*, 1983]. For other probability distributions, numerical techniques must be used. A numerical Monte-Carlo simulation technique is adopted here, as described in *Kuczera* [1999]. In particular, considering that the cost and the damage functions are independent on the set of parameters $\boldsymbol{\Theta}$, the order of integration can be changed and equation (11) can be rewritten as:

$$\begin{aligned} \widetilde{C}_{TOT}\,(q^* \mid c, d) &= \int_{\boldsymbol{\Theta}} \left(c \cdot q^* + \int_{q^*}^{\infty} d \cdot (q - q^*) \cdot p(q \mid \boldsymbol{\Theta}) \mathrm{d}q \right) \cdot h\,(\boldsymbol{\Theta})\, \mathrm{d}\boldsymbol{\Theta} = \\ &= \int_{\boldsymbol{\Theta}} C_{TOT}(q^* \mid c, d, \boldsymbol{\Theta}) \cdot h(\boldsymbol{\Theta}) \mathrm{d}\boldsymbol{\Theta}, \end{aligned} \tag{13}$$

Using equation (13), the numerical integration procedure is implemented according to the following main steps:

1. the vector $\boldsymbol{\Theta}$ of parameters is randomly sampled k times from the corresponding multivariate parameter pdf $h\,(\boldsymbol{\Theta})$, obtaining the parameters set $\{\boldsymbol{\Theta}_k, k = 1, \ldots, M\}$;

2. once the parameter sets are sampled, equation (8) is applied to each of them. A set of total expected cost functions is then obtained, one for each set of parameters (see Figure 3, dotted grey lines).

3. the C_{TOT} curves are averaged together, obtaining an average total expected cost function (see solid line in Figure 3);

4. the value of q^* corresponding to the minimum of the function is selected as the optimal design flood estimator in uncertain conditions, q^*_{unc}.

In step 1 of the Monte-Carlo procedure, the pdf $h(\mathbf{\Theta})$ of the distribution parameters is required. In general terms, $h(\mathbf{\Theta})$ depends both on the type of probability distribution function $p(q \mid \mathbf{\Theta})$, and on the parameters estimation technique. Here, the L-moments technique for parameter estimation is used [see e.g. *Stedinger et al.* [1993]]. In uncertain conditions, the application of the L-moments estimation technique is particularly convenient, as demonstrated by *Hosking and Wallis* [1997], because the pdf of the L-moments depends only weakly on the discharge pdf $p(q \mid \mathbf{\Theta})$: in fact, L-moments tend to be normally distributed even with small samples [*Hosking and Wallis*, 1997]. Once the pdf of the L-moments is available, the multivariate distribution of parameters can be obtained as a derived distribution (note that the relationship between parameters and L-moments is not linear). In terms of the numerical application this means that: i) firstly, the L-moments pdfs are obtained consistently with the available sample of data; ii) a family of k L-moment sets is randomly sampled from their pdf's; iii) the set of k parameter vectors $\mathbf{\Theta}$ required in the step 1 of the above-described procedure is obtained from the corresponding L-moments set ***, as derived distributions (note that the transformation is non-linear).***

For analytical convenience L-moments ratios are often used instead of L-moments: if $\bar{q}$ is the mean discharge, equal to the L-moment of order one, τ_2 (also defined as L-CV) is the ratio between the L-moment of the second order and $\bar{q}$ and τ_3, or L-CA, is the ratio between the L-moment of the third order and the L-moment of the second order. Here, simple formulas reported in *Viglione* [2007] were used to obtain the pdfs of the L-moments ratios $\bar{q}$, τ_2, τ_3. More in detail, τ_2, τ_3 are described by a bivariate Normal distribution because τ_2, τ_3 are correlated, while $\bar{q}$ is described by a univariate Normal distribution because $\bar{q}$ is typically independent on τ_2 and τ_3 [further details are reported in *Hosking and Wallis* [1997]; *Elamir and Seheult* [2004] and *Viglione* [2010]].

4.2. Comparison between standard and UNCODE flood quantiles

The UNCODE flood estimators q_{unc}, computed as described above, can be quite different from the values obtained from the Standard design flood estimators q_T, called here the Standard ones. The differences between the two can be assessed in terms of deviation of their confidence probability, CP, a non-exceedance probability associated to q_T and q_{unc}, ***computed on the confidence bands. Confidence bands are calculated according to the following steps: i) L-moments ratios are sampled from their relative probability distribution functions; ii) from each samples, parameters are estimated, imposing a specific probability distribution function (derived distribution); iii) quantiles are computed for a given exceedance probability from each samples (quantile extraction), so that for each exceedance probability (or return period, as reported in the figures) an empirical probability distribution function of quantiles is obtained. Derived distributions estimation and quantile esxtraction are both non-linear transformations. Note that the L-moments ratios, and consequently the parameters and the applied probability distribution function used to derive the confidence bands are exactly the same as those applied to calculate the optimal design flood estimators,*** q_{unc}***, whose sampling hypothesis and procedure have been described in section***

4.1. Otherwise, the confidence bands and the optimal design flood estimators reported over them would not be consistent to each other.

The meaning of CP can be explained by an example: in Figure 4, panel a), confidence bands computed for a sample of data are displayed. For a given return period, say $T = 500$ years, the Standard design flood quantile is estimated (as displayed by the squared point). The UNCODE estimate is represented by the the triangle-shaped point, and the pdf of the flood quantile is depicted on the right (see also Figure 1). Considering the position of the Standard and of the UNCODE design floods on the flood quantile pdf (Figure 4 panel b), the CP for each estimator is defined as its non-exceedance probability computed on the quantile flood probability curve. The comparison between the two estimates can be assessed through a coefficient γ defined as $\gamma = (CP_{unc} - CP_T)$ where CP_{unc} and CP_T are respectively the confidence probability of the UNCODE and of the Standard estimates as illustrated in Figure 5, panel a, b and c. Since the L-moments pdfs are normal, the Standard design flood estimate q_T converges towards the median and its CP is thus always equal to 0.5. Therefore, the domain of γ spans from -0.5 to $+0.5$, where the positive values of γ indicate UNCODE estimates larger than the Standard ones.

5. Application to real-world flood data sets

The procedure described in the preceding paragraphs has been applied to a set of 10 series of annual maxima of flood peaks from sub-catchments of the Po river located in the North–West of Italy. In Table 1 some basic information about the considered flood records are reported. A 3-parameters Lognormal probability distribution has been first used to fit the flood records, as suggested in previous studies [*Laio et al.*, 2011], but the Generalised Extreme Value (GEV) pdf has also been applied to check if the choice of the probability distribution function plays a significant role in determining the outcome of the procedure. Simulations are based on the selection of 5 return periods $\{T = 50, 100, 200, 500, 1000$ years$\}$ and $k = 10000$ sets of sampled L-moments ratios, generated from the corresponding probability distribution functions (see section 4.1).

The model is at first applied in non-uncertain conditions: ~~this isachieved by setting to 0 the standard deviation of the L-moment ratios distribution functions; no dispersion is then obtained and the Standard and UNCODE design flood estimators converge, as expected, to the same value, located on the median curve of the confidence bands (dotted line in Figure 3). This trivial case is useful to check the accuracy of the numerical procedure.~~

This is achieved by setting converging to 0 ***(i.e. meaning a value which can be considered small if compared to the values of discharge data applied, around 3 or 4 orders of magnitude inferior) the standard deviation of the L-moment ratios distribution functions so that no dispersion is obtained; in other word, this means that parameter uncertainty is not taken into account. This trivial case is useful to check the correctness of the numerical procedure, which is expected to converge to the standard qT, as analytically demonstrated by equation (8) and (9). As expected, the Standard and UNCODE design flood estimators converge to the same value, located on the median curve of the confidence bands (dotted line in Figure 4)***. When uncertainty is fully taken into account, the procedure produces only positive γ values, regardless of the return period T. This indicates that the uncertainty-compliant design flood is systematically larger than the Standard value, ***consistent to what is reported by and Stedinger [1983], who demostrated the non central t distribution of a quantile***

estimate with normal distribution and by Arnell [1989] who did numerical experiments given the assumpion of non-central t distribution. This can be recognized from Figure 6, where the estimated coefficients γ for 5 different return periods and for each series are reported. The solid black line is the mean value of γ obtained from the 10 series for each return period; it can be seen that γ increase quite linearly (in the semi-logarithmic scale) for increasing return period T. The increment is marginal when low return periods are considered, but becomes critical for return periods larger than 100 years.

When the GEV distribution is considered all γ values remain positive, regardless of the return period T, with γ increasing for increasing return period T (except for the case of the Dora Riparia a Oulx for $T = 50$ years); Figure 7 reports the results obtained with the GEV distribution for each series, together with the mean curve (solid black line).

Solid black lines in Figure 6 and Figure 7 show that, in average, coefficients γ are positive both for LN3 and GEV (with a range spanning from 0.05 to 0.24 for high return periods); the increment is almost linear with T in semi-logarithmic scale, with higher slope in the case of the $LN3$ distribution.

5.1. Effect of sample length

In evaluating the results of the above procedure, it is quite important to consider the different content of information that resides in hydrological records of different length. To investigate the effects of sample size in the deviation of q_{unc} from q_T, a numerical experiment has been made by using LN3 and GEV distributions on the longest available flood series, Dora Baltea at Tavagnasco ($n = 82$). The effect of sample size has been investigated by considering estimates obtained by hypothetical shorter samples. 6 different lengths $\{n = 82, 70, 55, 40, 25, 10\}$ were considered. The samples preserve the same mean and L-moments ratios of the original series, but the related standard deviation increases with decreasing length of the sample, following *Viglione* [2007].

The UNCODE and Standard design flood estimates for 5 return periods T were then computed based on each sub-sample characteristics. Analysis of results show that the increased variance from short samples determines an increment in the value of the γ coefficient for any return period T. As an example, for the return period $T = 100$ years the coefficient γ is 0.1 for the full-length series and increases to 0.145 for the sub-sample with $n = 25$. These results are displayed in Figure 8. The fact that γ increases with decreasing n implies that the UNCODE estimator is very far apart form the Standard one when small samples are considered: in fact, the distance between q_T and q_{unc} increases both because the UNCODE estimator moves away from the median value (in fact, γ increases) and because the distribution of design values spans a larger range, due to the greater uncertainty (i.e., even when γ is fixed, $q_{unc} - q_T$ will be larger in smaller samples). This is apparent in Figure 9, where the confidence bands of two different sub-sampled series, $n = 82$ and $n = 25$, are depicted, to provide a clearer context in which the difference between q_T and q_{unc} is obtained. Note that the design flood increases of a 1.5 factor for T=100 years when n=25.

6. Discussion and conclusions

The present work considers the effect of the parametric uncertainties in the estimation of design flood quantile. A "design" model has been developed in which parametric uncertainties and cost-benefit analysis are integrated in the standard flood frequency analysis. It has been demonstrated that the standard flood frequency analysis estimator and the design flood estimator provided by the cost-benefit analysis (without uncertainty) with linear damage and cost functions are equal when the ratio between the slope of the damage and the cost function is equal to T. This analytical result is a key concept in the whole subsequent inference procedure: it demonstrates that these two techniques, the standard flood frequency analysis and the cost-benefit analysis, are totally equivalent when uncertainty is neglected; moreover, the cost-benefit analysis does not introduce any further uncertainty in the flood frequency analysis. In the presence of uncertainty, the economic-driven approach is then used to obtained a design flood estimator which corresponds to the minimum of the total cost (where the total cost is the sum of the costs to build the hydraulic infrastructure and the damages which might occur in case of overflow). The devised procedure leads to the UNcertainty COpliant DEsign value that can be quite different from the Standard one. To assess the displacement in the design values induced by uncertainty practical applications have been implemented for 10 time series of Italian catchments. Results show that the UNCODE design flood estimates are systematically larger than the Standard ones, with the difference becoming more and more substantial for high return periods ($T > 100$ years). This suggests that the standard flood frequency procedures may lead to underestimated design floods. Results are negligibly influenced by the type of probability distribution functions considered, while sample length plays a role: short sample length moves the UNCODE flood estimator to even larger values, recasting under a new light the role of data availability in flood frequency analysis. Indeed, a scarce data availability does not only increase the amplitude of the confidence bands, but also moves to larger values the design value minimising the expected cost, i.e. the UNCODE estimator. ***Due to the flexibility of the UNCODE approach,***

many improvments to the models, such as non-linear cost-damage functions, a prior information and non-stationarity analysis (Salas and Obeysekera [2014]), could be further investigated in the future.

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Table 1. Summary of the 10 series used in the case study.

Station number	**Name**	Acronym	**Record length**[n]	**A** [km^2]	H_m[m]
1	Dora Baltea a Tavagnasco	DBATA	82	3320.85	2087
2	Tanaro a Farigliano	TANFA	69	1502.15	945
3	Stura di Lanzo a Lanzo	SLALA	64	578.31	1780
4	Bormida a Spigno a Valla	BSPVA	52	68.46	468
5	Dora Riparia a Oulx	DRIOU	43	260.04	2164
6	Rutor a Promise	RUTPR	33	45.76	2525
7	Corsaglia a Presa Molline	CORPM	25	89.31	1525
8	Po a Carignano	POCA	16	3955.59	1101
9	Belbo a Castelnuovo Belbo	BELCA	13	420.75	372
10	Malone a Brandizzo	MALBR	10	333.37	439

[a] Footnote text here.

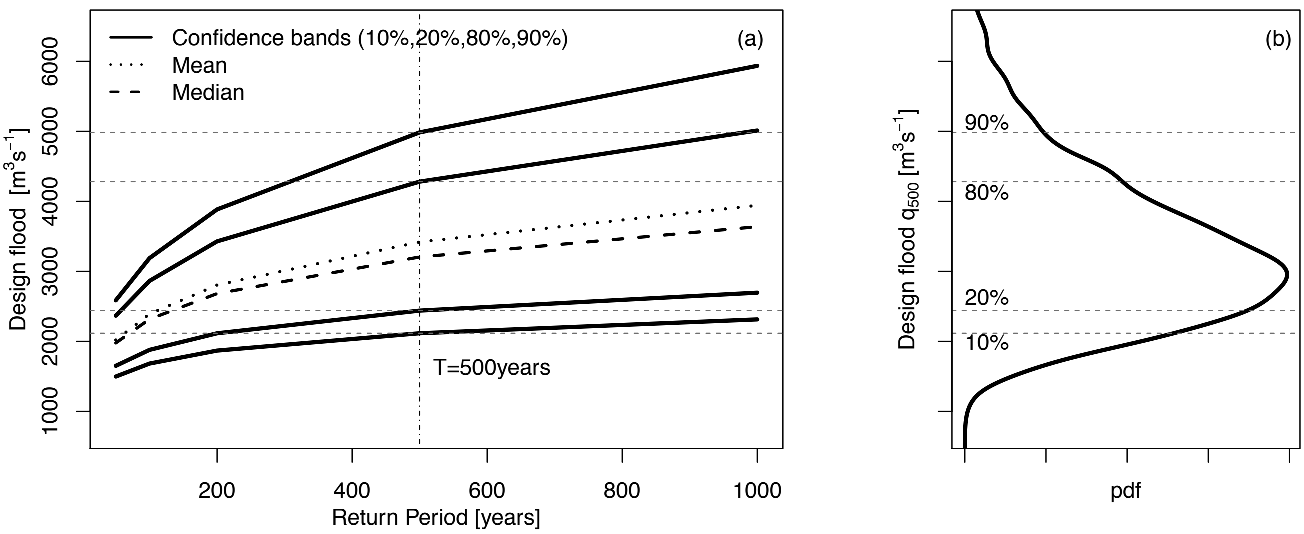

Figure 1. Uncertainty evaluation and definition of the confidence bands. Suppose to fix a return period $T = 500$ years (panel a): in the case uncertainties are accounted for, it is possible to obtain a probability distribution function of design flood estimator q^* (panel b) instead of a single value for the specific T.

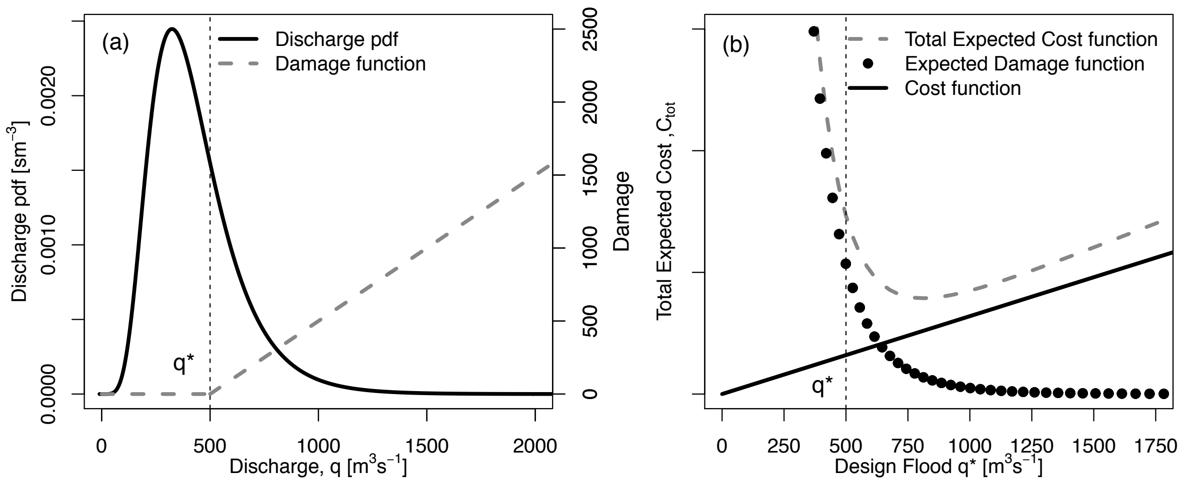

Figure 2. Panel a) Construction of the expected damage function: suppose to fix the value of the design flood estimator $q^* = 500$ m^3/s: by doing this, the damage function is defined according to equation (2); the integral of the product of the damage function and the flood probability distribution function is equal to the single value of the expected damage function, ED corresponding to $q^* = 500$ m^3/s, as presented in equation (4). Panel b)The total expected cost function, C_{TOT} is built as the sum of the cost function, CF, and the expected damage function, ED.

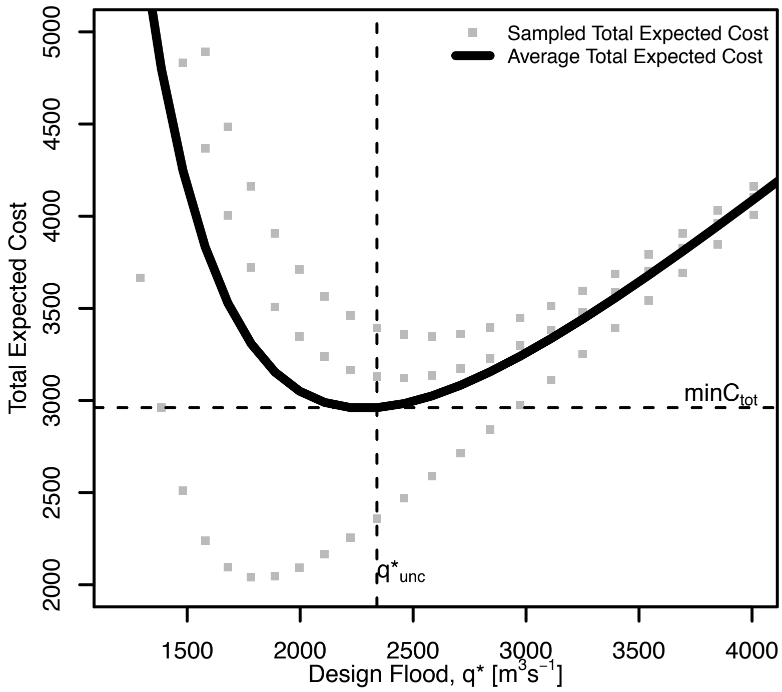

Figure 3. Numerical implementation of the method to obtain the UNCODE flood estimator. Each of the dotted curves represent a total expected cost function obtained from different sets of parameters, $\boldsymbol{\Theta_k}$, randomly sampled from the relevant distribution function. The solid curve stands for the average expected cost function. The minimum of the curve is the UNCODE estimator, q^*_{unc} (only three sampled total expected cost functions are reported here out of the 10000 used).

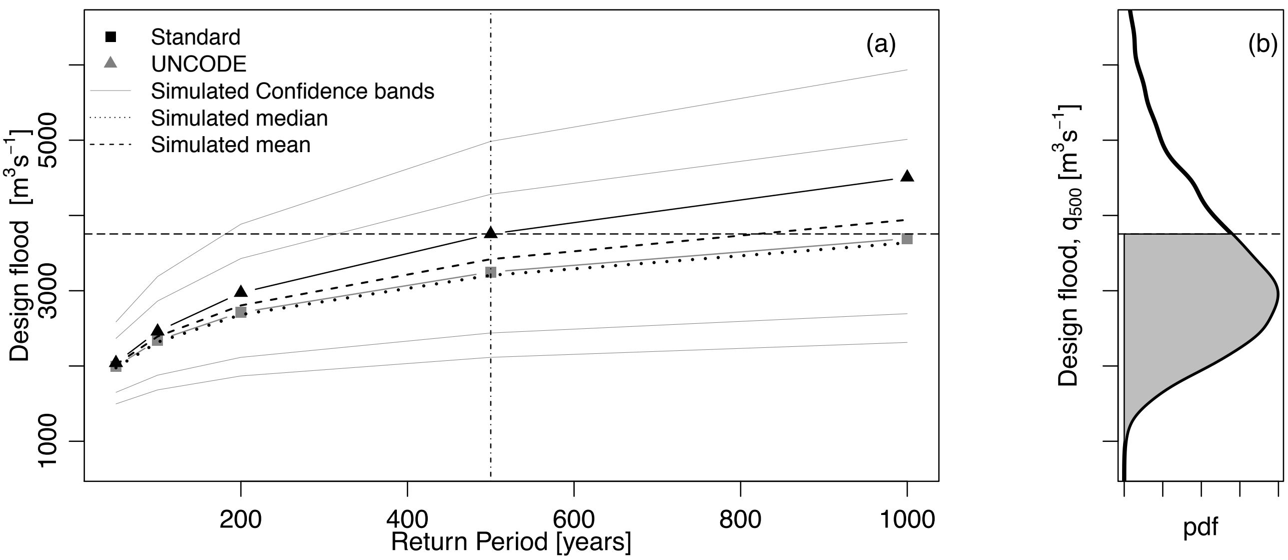

Figure 4. CP definition: given a return period $T = 500$ years, and the corresponding q^*_{unc} estimator, CP is the non-exceedance probability of the UNCODE estimator, measured on the design flood probability distribution (coloured area in panel b). Note that the CP for q_T is equal to 0.5.

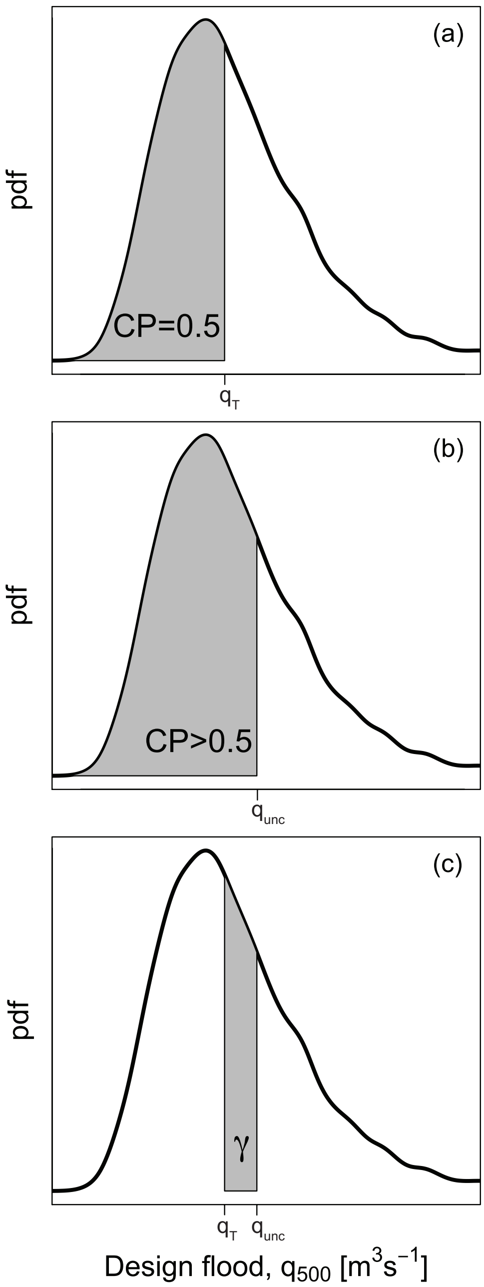

Figure 5. Standard design flood estimators corresponds to a confidence probability (as depicted in figure4) equal to 0.5 (panel a); UNCODE estimators present a value of CP larger than 0.5 (panel b); the difference between Standard design flood estimators and UNCODE estimators can be appreciated using the coefficient γ (panel c).

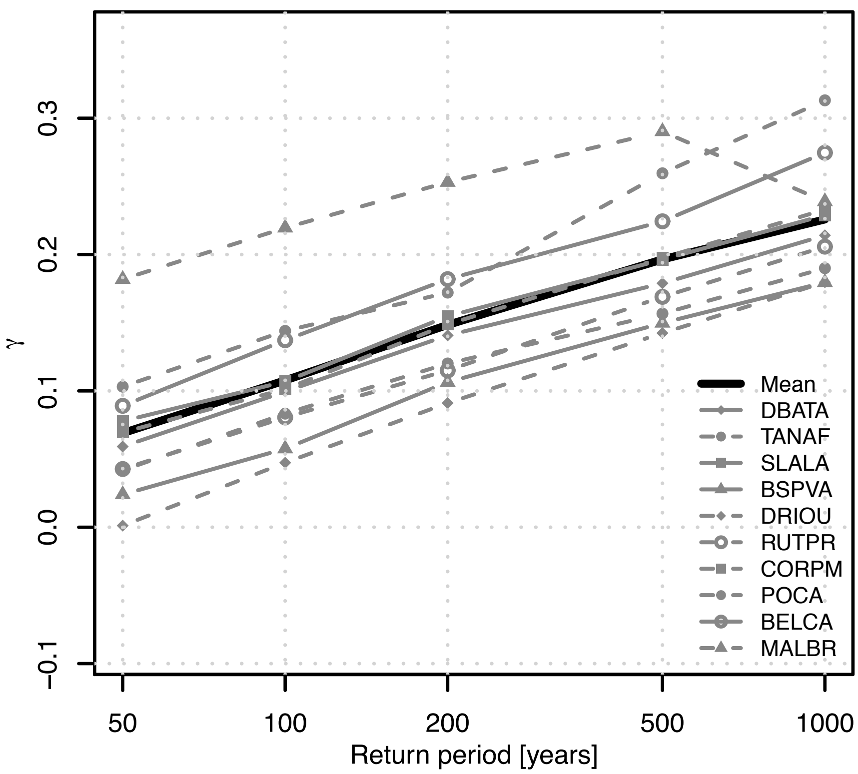

Figure 6. Coefficient γ calculated on the 10 series for the LN3 pdf. The black solid line is obtained by averaging the 10 series.

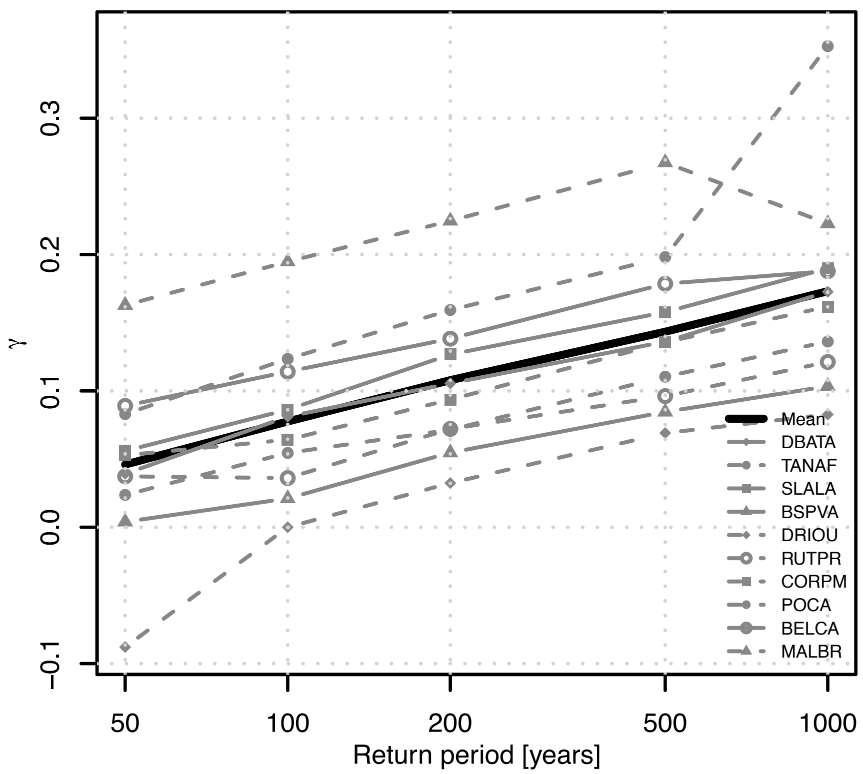

Figure 7. Coefficients γ calculated on the 10 series for the GEV pdf. The black solid line is obtained by averaging the 10 series.

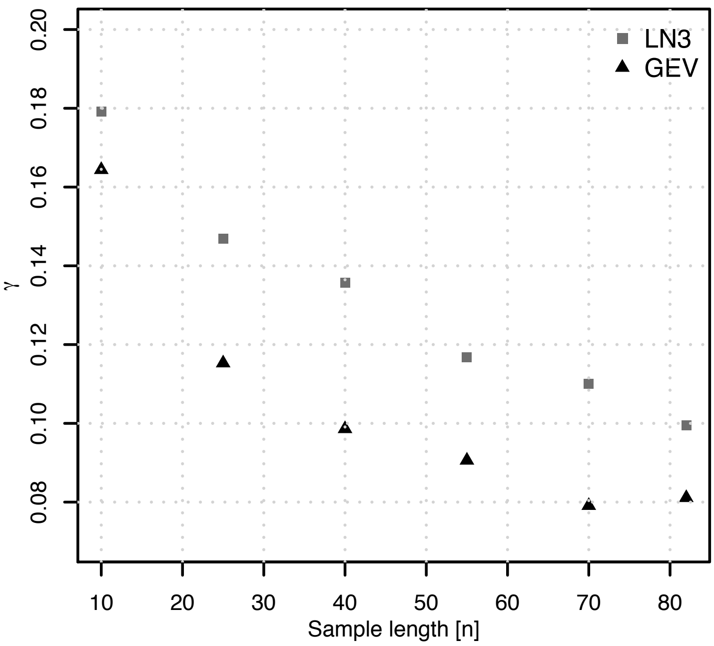

Figure 8. Dora Baltea at Tavagnasco confidence coefficient in case of sub-sampling. Sub-sample lengths are n=10,25,40,55,70,82. Each point represents the value of the confidence coefficient of a specific sub-sample for a return period $T = 100$ years.

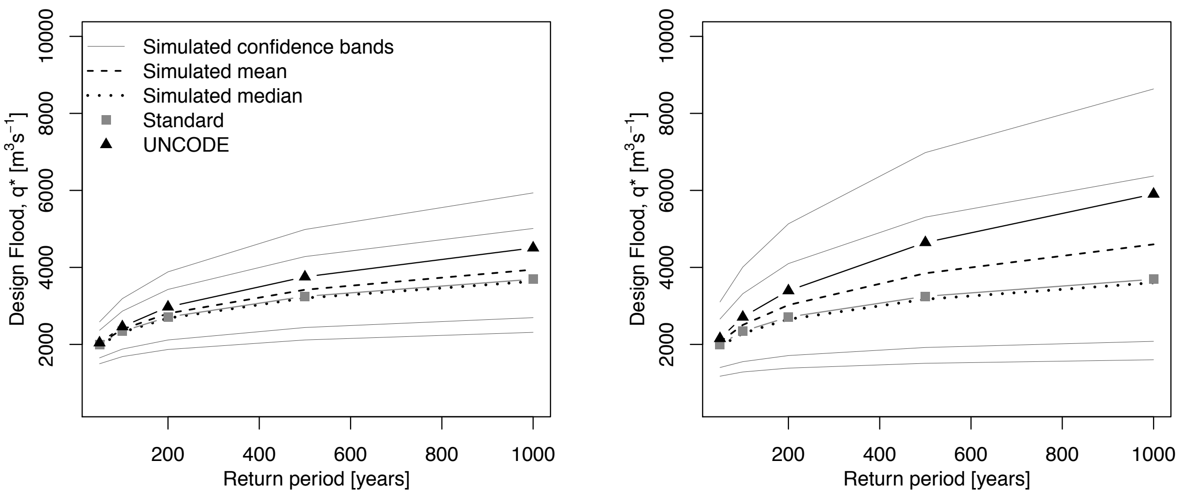

Figure 9. Influence of the sample length on the confidence bands: the figure depicts the different dispersion around the median of the confidence bands in the case of full sample, $n = 82$ (on panel a) and $n = 25$ (on panel b) for the Dora Baltea at Tavagnasco record.