Some Considerations on Univariate and Multivariate Streamflow Simulation Methods

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Abstract: Streamflow simulation over time intervals much longer than observation periods requires the evaluation of the quality of the reproduction of observed statistics of time series. Since much of the non linearity of the rainfall-runoff transformation is in the conversion of rainfall to effective rainfall (ER), two approaches with fundamental differences in the ER estimation are put under comparison, to highlight pros and cons of univariate and bivariate methods. We propose a comparison of IHACRES and Shot Noise models, which involve direct and inverse estimates of ER at the daily time scale. The comparison is carried out in terms of assessment of inconsistencies in evaluating ER by direct and inverse estimates, both in temperate and in alpine basins. Additional comparisons regard the reproduction of direct runoff with the two approaches. The results demonstrate that relatively few improvements are obtained by considering a bivariate streamflow simulation method, i.e. by using the rainfall time series for streamflow modelling.

1. Introduction

Streamflow simulation over time intervals much longer than observation periods is an evergreen practice in hydrology. From a technical viewpoint, the quality and physical soundness of existing models is far sufficient for obtaining good results in standard applications (e.g., Salas, 1993), i.e. when enough rainfall and runoff data are available.

To grasp the physical mechanisms acting in streamflow formation, many models are bi-variate or multi-variate (see e.g., Farmer et al, 2003; Manfreda et al., 2005), requiring at least a few years of contemporary rainfall and runoff records. This data requirement is necessary for providing the possibility of model verification but, in the literature, very little attention has been paid so far to procedures for quality assessment of the generated data series. The debate on the parsimony and efficiency of simulation models is still in progress and requires additional research in this field (see Claps et al., 2005). In fact, the presence of refined simulation software packages can give the average user false certainties about the quality of simulated data.

As a matter of fact, even in the presence of sufficient and good-quality data, the use of multi-variable simulation models does not guarantee the best quality of simulated series. In this sense, univariate models are still in place in providing instruments for parsimonious streamflow simulation. This characteristic becomes especially precious when the uncertainties in the areal rainfall measurements are high, as it usually happens in the mountainous environments.

To proceed with the discussion on model efficiency, rainfall-runoff modelling at the daily scale has been considered, in the perspective of application of univariate (Shot Noise, Murrone et al., 1997) and of bi- (or tri-) variate (IHACRES, Jakeman et al., 1990) well-established simulation models. The rainfall-runoff transformation processes have been examined considering two macroscopic steps: the rainfall to effective rainfall (ER) and the ER to runoff transformations. This allows one to attempt a comparison between models that have a slightly different structure and that use different types of variables. Moreover, because the Shot Noise model provides an inverse estimation of the effective rainfall from streamflows, the comparison has two objectives: i) to provide information on the nature of errors that accompany rainfall measurement in mountain basins, because inversely estimated ER must necessarily be consistent with measured runoff; ii) to select clues of uncertainty of the areal rainfall information, in order to consider the relative advantages of the inverse estimation .

2. Description of IHACRES and Shot Noise models

In the present section the basic characteristics of the two considered models are briefly reviewed, with special attention to the estimation of effective rainfall in the models. Effective rainfall can be defined as the part of the rainfall that actually reaches the basin outlet, and, in impervious basins, it can be derived from total rainfall by subtracting the evapotranspiration amount (see Mls, 1980; Sirangelo and Murrone, 1996)

Direct or inverse estimates of ER can be carried out. A direct estimate of ER is obtained evaluating evapotranspiration, generally using temperature data. Considering watersheds as linear systems, ER represents the argument of the convolution integral, and must have the same average as runoff. As such, it can be derived through deconvolution of runoff, given the system response function (inverse estimate). Examples of two models employing direct or inverse strategies for ER estimation are provided in the following.

In IHACRES (Jakeman et al., 1990) the rainfall-runoff transformation is obtained with two modules: a non linear loss module, that transforms precipitation to effective rainfall by considering the (direct -if available- or indirect) influence of temperature, and a linear module, based on the classical convolution of the effective rainfall by the unit hydrograph (UH), that produces the total streamflow. The non-linear loss module involves the calculation of an index of catchment storage *s(t)* for every time step *t*, based upon a negative exponential weighting of precipitation and temperature:

$$s(t) = \frac{r(t)}{c} + \left(1 - \frac{1}{\tau_w[T(t)]}\right) \cdot z^{-1} \cdot s(t) \quad (1)$$
$$\tau_w[T(t)] = \tau_w \cdot e^{[0.062 \cdot f \cdot (20 - T(t))]}$$

In (1), *s(t)* is the catchment storage index, $\tau_w[T(t)]$ is a variable controlling the rate at which the catchment wetness index $s(t)$ decays in the absence of rainfall, τ_w is the value of $\tau_w[T(t)]$ at T=20°C, c is a parameter chosen to constrain the volume of effective rainfall to equal runoff, f is a temperature modulation factor, z^{-1} is the backward shift operator. The effective rainfall $ER(t)$ is computed as the product of total rainfall *r(t)* and the storage index *s(t)*,

$$ER(t) = s(t) \cdot r(t) \quad (2)$$

and then convolved with the unit hydrograph of the two-reservoirs-in-parallel linear system,

$$h(t) = \frac{\nu_q}{\tau_q} \cdot e^{-\frac{t}{\tau_q}} + \frac{\nu_s}{\tau_s} \cdot e^{-\frac{t}{\tau_s}} \quad (3)$$

The above relation is a function of the basin dynamic response characteristics (DRCs) (Littlewood *et al*, 2003) ν_q and ν_s (relative volumetric throughputs for quick and slow flow) and τ_q and τ_s (characteristic decay time constants for quick and slow UHs), and ot the time step t.

Looking at the same system from the point of view of univariate modeling, streamflow can be considered as generated by a Shot Noise process, (Murrone *et al.*, 1997) that is a filtered sequence of independent and instantaneous pulses. It can be represented as:

$$q(t) = \sum_{i=1}^{N_t} ER_i \, h(t - \tau_i) \quad (4)$$

where t is time, q is runoff, N_t is the total number of occurrences up to time t, ER_i is the intensity of the i-th pulse, $h(t-\tau_i)$ is the response function, and τ_i is the time of occurrence of the i-th pulse.

The univariate Shot Noise model devised by Murrone et al. (1997) is based on the observed runoff series, and reconstructs the ER sequence by deconvolution, without requiring any information on the rainfall and temperature forcings. This inverse procedure requires a preliminary identification of the effective rainfall events, e.g. in correspondence to the days when the streamflow increases. This initial identification of the occurrences allows one to evaluate the ER_i amounts by deconvolution while estimating the parameters of the response function.

The basin response function used in the model differs from that of the IHACRES because of the presence of a zero-lag additive term, c_0, representing surface runoff,

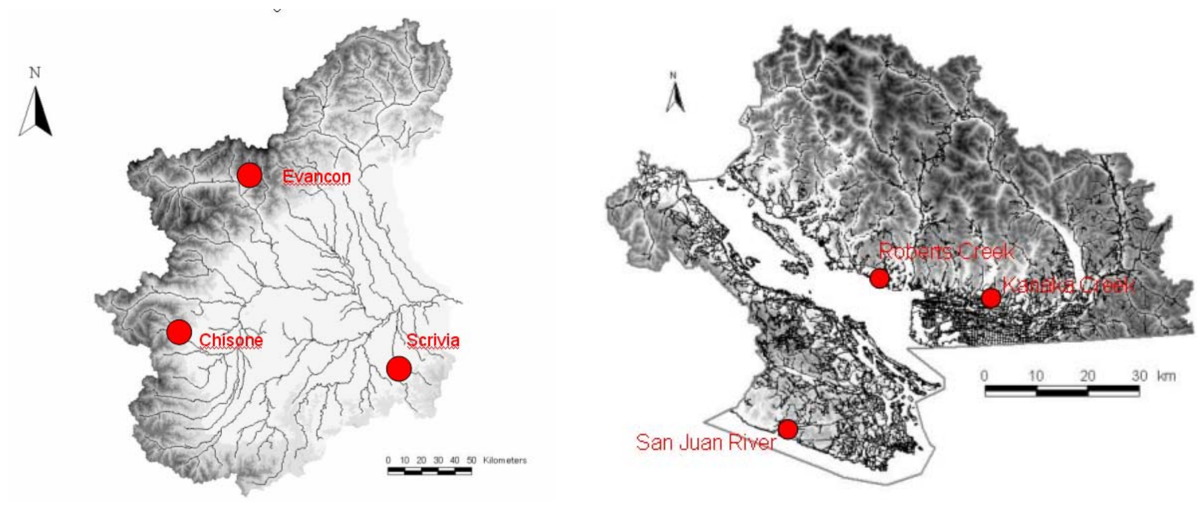
$$h(t) = c_0 + \frac{c_1}{k_1} \cdot e^{-\frac{t}{k_1}} + \frac{c_2}{k_2} \cdot e^{-\frac{t}{k_2}} \quad (5)$$

while the other terms can be easily associated with the analogous ones in the IHACRES UH expression.

Parameters c_i and k_i are estimated by minimizing the sum of the quadratic distances between observed and reconstructed data. This requires an iterative procedure, because the final response function is not known when evaluating the first effective rainfall series by deconvolution. Anyway, no calibration or adjustment is needed to equalize the runoff and ER averages: this is guaranteed by the deconvolution procedure itself.

3. Application: effective rainfall estimation

The comparison between the IHACRES and Shot Noise models is carried out in terms of effective rainfall and direct runoff estimation. Six basins have been examined in the application (see Figure 1): three rainfall-driven coastal watersheds of British Columbia (Canada) and three basins located in northern Italy, respectively a temperate, a "transition" and a "pure" alpine watershed. The respective watershed nature can be recognised by the value of the average elevation, listed in Table 1 with other watersheds characteristics and with the cross-correlation coefficient between observed rainfall and runoff (R). In temperate watersheds precipitation series show a strong correlation with observed runoff.



(a) (b)

Figure 1. Location of the considered watersheds in the North-West of Italy (a) and in the South west of British Columbia (Canada) (b).

In alpine environments, in contrast, runoff derives also from snowmelt and precipitation values are often affected by significant errors. As a consequence, the cross correlation coefficient declines to very low values (0.15 for the Evançon river at Champoluc). In this sense R can be assumed as an index to discern between temperate and snowfall-driven basins.

The parameters to be set in the basin response functions, (3) and (5), are reported in Table 2 for each of the 6 basins. Note that for the Evançon river at Champoluc the calibration procedure of the IHACRES method does not converge, and it is therefore impossible to find the parameter values.

The lack of correlation between rainfall and runoff in alpine basins can be a clue of possible problems in the direct estimate of effective rainfall. This can be studied by considering the time series of occurrences of direct and inverse ER estimates, starting from basins in temperate regions. An example of the two time series, for the case of the Scrivia river at Serravalle, is provided in Figure 2. The superposition of the two estimated ER time series shows that the application of the direct method implies a proliferation of ER events (grey bars in Figure 2): all rainfall events in IHACRES are supposed to produce an effective rainfall impulse (see Equations (1) and (2)). In contrast, when an inverse ER estimation method is adopted, only few, very intense ER events are retained (black bars in Figure 2), as a consequence of the application of a threshold filter in the Shot Noise procedure (see Murrone et al. (1997) and Claps et al. (2005) for details). The conclusions drawn from the visual inspection of Figure 2 can be generalized with a correlation analysis of the time series of directly and inversely estimated ER occurrences.

Table 1. Watersheds main characteristics (area, mean elevation, average annual rainfall, average annual discharge) and cross correlation coefficients R and Q (see text for details).

	Area [km^2]	Mean Elev. [m]	r [mm/y]	y [mm/y]	R	Q
San Juan Riv. @ Port Renfrew	580	663	3452	2604	0.85	0.84
Kanaka Creek @ Webster Corners	48	460	1807	1818	0.75	0.78
Roberts Creek @ Roberts Creek	33	697	1383	993	0.6	0.78
Scrivia @ Serravalle	611	695	1389	827	0.67	0.71
Chisone @ S.Martino	580	1730	1058	694	0.45	0.29
Evançon @ Champoluc	102	2631	1084	977	0.15	*

[* The IHACRES method provides unreliable results]

Table 2. IHACRES and Shot Noise parameters calibrated for three Canadian and three Italian basins

Basin	IHACRES		SHOT NOISE	
San Juan Riv. @ Port Renfrew	τ_q	0.76	k_1	1.86
	τ_s	53.15	k_2	70.63
	ν_q	0.47	c_1	0.45
	ν_s	0.53	c_2	0.41
			c_0	0.14
Kanaka Cr. @ Webster Corners	τ_q	1.69	k_1	1.28
	τ_s	89.20	k_2	61.63
	ν_q	0.67	c_1	0.38
	ν_s	0.33	c_2	0.44
			c_0	0.18
Roberts Cr. @ Roberts Creek	τ_q	3.47	k_1	1.62
	τ_s	114.13	k_2	61.48
	ν_q	0.54	c_1	0.35
	ν_s	0.46	c_2	0.50
			c_0	0.15

Basin	IHACRES		SHOT NOISE	
Scrivia @ Serravalle	τ_q	1.00	k_1	2.14
	τ_s	17.84	k_2	45.66
	ν_q	0.49	c_1	0.34
	ν_s	0.51	c_2	0.54
			c_0	0.12
Chisone @ S.Martino	τ_q	1.29	k_1	6.31
	τ_s	29.15	k_2	220.03
	ν_q	0.20	c_1	0.28
	ν_s	0.76	c_2	0.66
			c_0	0.06
Evançon @ Champoluc	τ_q	*	k_1	22.29
	τ_s	*	k_2	3507.80
	ν_q	*	c_1	0.64
	ν_s	*	c_2	0.30
			c_0	0.06

[* The IHACRES method provides unreliable results]

The time series of the occurrences are first reverted into binary time series, by attributing a value 1 to the days when an effective rainfall occurs, and a value 0 to the days when it does not. Standard statistical tools can then be applied to compare the two binary series: a modified form of the cross-correlation coefficient, which is suited for an application to binary time series, is the Yule's Q coefficient (Goodman and Kruskal, 1979). The Yule's Q gives a measure of the proximity of the two series based on a 2x2 contingency table:

	0	1
0	*N1*	*N2*
1	*N3*	*N4*

In the table, N1 represents the frequency of occurrence, in the two series, of the (0,0) couple of values, N2 of the (0,1) couple, N3 of (1,0) and N4 of (1,1). Accordingly, the Yule's Q is written as:

$$Q = \frac{N1 \cdot N4 - N2 \cdot N3}{N1 \cdot N4 + N2 \cdot N3} \tag{6}$$

and varies between −1 and 1, with large values implying highly correlated binary series.

The Yule's Q values computed in relation to ER series, estimated by IHACRES and Shot Noise, are reported in the last column of Table 2. The results show that in basins with rainfall-driven streamflows, characterized by a high correlation coefficient between observed rainfall and runoff, it is possible to achieve a reasonable synchronicity between directly and inversely computed ER series, as proved by the high value of the Q coefficient. On the contrary, in alpine environments direct and inverse ER estimates are poorly correlated. For the Chisone basin the Yule's Q coefficient is very low and for the Evançon basin direct ER estimates become even unreliable.

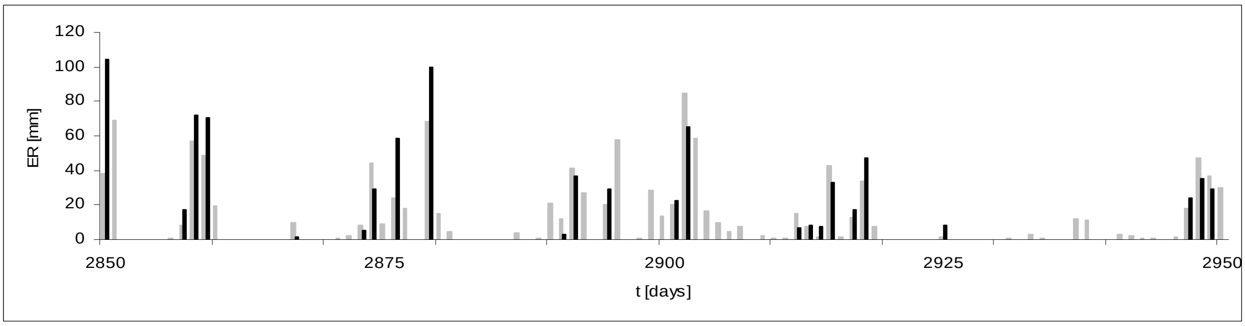


Figure 2. Directly (grey) and inversely (black) estimated ER time series for the Scrivia watershed.

The large Q values in temperate basins represent an interesting result, supporting the reliability of the inverse method for the ER estimate (for this typology of basins, the direct ER estimates can in fact be supposed to be rather robust).

In contrast, in alpine basins the low Q values can be a symptom of a lack of consistency of either the direct or the inverse method (or possibly of both). We note that in alpine environments the precipitation measures are often very unreliable, with mean annual discharge values often significantly larger than mean annual observed precipitation. This undermines the credibility of direct ER estimates. Moreover, solid and liquid precipitation surely have a markedly different effect on ER production: this implies that direct estimation of ER would require a different model structure in alpine basins, while inverse estimation at least allows one to balance, on the average, effective rainfall and runoff. Based on these results, the indirect ER estimation apparently presents a significant advantage over the direct one.

4. Application: direct runoff estimation

As mentioned in Section 2, the IHACRES and Shot Noise models have a rather similar structure in terms of the adopted response function. However, the presence of the term c_0 in the Shot Noise response function hampers a direct comparison between the parameter values reported in Table 2. A valuable alternative is to compare the obtained results in terms of "direct runoff" (DR), which represents the rainfall quota that contributes to the streamflow formation within the same unit time interval considered for rainfall, that in this case is of one day (see Singh and Aminian, 1986; Pilgrim and Cordery, 1993).

The direct runoff series is obtained by using, in the convolution integral, a subset of the response function, obtained by setting $t=1$ (day) in equations (3) and (5),

$$h_{dr}(t) = \frac{\nu_q}{\tau_q} \cdot e^{-\frac{1}{\tau_q}} + \frac{\nu_s}{\tau_s} \cdot e^{-\frac{1}{\tau_s}} \qquad (7)$$

$$h_{dr}(t) = c_0 + \frac{c_1}{k_1} \cdot e^{-\frac{1}{k_1}} + \frac{c_2}{k_2} \cdot e^{-\frac{1}{k_2}} \qquad (8)$$

Unlike ER, the direct runoff is a continuous variable, for which the meaning of a correlation analysis can be rather elusive, possibly giving too much weight to the prevailing low-flow values. Anyway, visual inspection allows one to make some considerations.

Figure 3 shows the computed IHACRES (positive axis) and Shot Noise (negative axis) DR time series, surmounted by the observed streamflow, for the case of the Scrivia river at Serravalle. Figure 4 shows the scatter plot of the Shot Noise versus the IHACRES direct runoff for the same watershed, represented in bilogarithmic scale. In this case, the agreement between directly and inversely estimated DR is quite good, except for the peak values, where the direct method tends to underestimate the discharge entity. Also the scatter plot in Figure 4 demonstrates the good agreement between the two DR estimates, with the previously mentioned exceptions in correspondence to the peak values. The analysis of the same diagrams for the other river basins confirms these findings, even if the similarity between the two series tends to decline when moving from temperate to alpine basins. Also the analysis of direct runoff therefore supports the conclusion that univariate streamflow simulation methods can be better than bivariate models in terms of quality of the obtained results.

5. Conclusions

The characteristics and performances of two streamflow simulation models of different structure have been analysed to highlight issues related to the identification and estimation of phenomenological and conceptual components of the rainfall-runoff transformation. The Shot Noise and IHACRES models have a similar structure in terms of (linear) effective rainfall to runoff transformation, but the ER series is obtained by inverse estimation in the former and directly from rainfall in the latter model. The effects of these analogies and differences have been examined in a comparative application to six daily discharge time series from basins located in different climatic contexts, three in Italy and three in Canada.

The application has demonstrated that the models exhibit a similar behaviour in temperate climates, in terms of values of conceptual parameters and characteristics of the estimated effective rainfall. As one moves from temperate to alpine basins the reliability of areal rainfall weakens and the role of snow in moderating runoff increases, so that the estimates of parameters, the identification of events and the magnitude of ER differ more and more. None of the two models has a specific module to deal with the effect of snow accumulation and melting. However, the features of the Shot Noise model (assurance of mass conservation, objective evaluation of ER from runoff) produce a more reliable representation of the streamflow process, in particular for basins in a transition (from temperate to alpine) environment. The reconstruction of the direct runoff time series also demonstrated to be more accurate when using the inverse estimation method, even in temperate climates, owing to its independency on measured rainfall and to the presence of a threshold filter. This filter, if applied to IHACRES would prevent it from producing runoff for each nonzero value of rainfall. The above indications suggest that a combined use of direct and inverse streamflow modelling could be very useful in transition and cold climates.

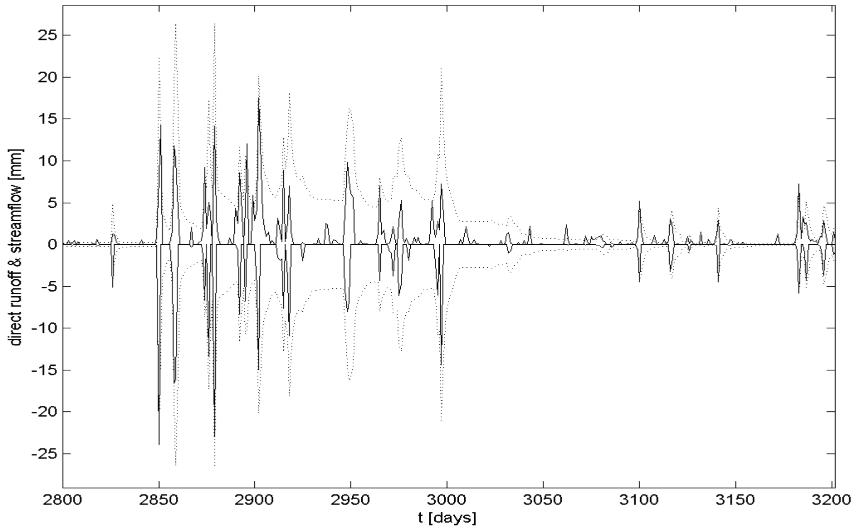


Figure 3. IHACRES (positive axis) and Shot Noise (negative axis) direct runoff, surmounted by the observed (dotted) streamflow for the Scrivia river basin.

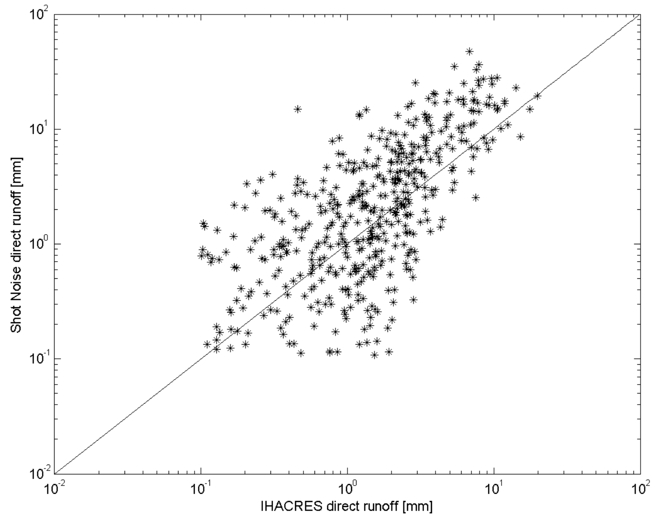


Figure 4. Shot Noise versus IHACRES direct runoff estimates for the Scrivia watershed (note the log-log scale).

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